As the title suggests, these notes are meant to be a supplement to the lecture on inductive and deductive reasoning. An argument is composed of a series of premises or supposed facts, and a conclusion which should be derived from the premises. The textbook gives formal definitions of the two types of arguments. An inductive argument makes a case for a general conclusion from more specific examples. A deductive argument makes a case for a specific conclusion from more general premises. These are nice definitions, but in practice it is difficult to determine if a specific argument is inductive or deductive.

Consider the following situation: Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice and also participated in antinuclear rallies. Rank the following possibilities from 1 (most likely) to 5 (least likely):

1. Linda is a teacher
2. Linda works in a bookstore and takes yoga classes.
3. Linda is a bank teller.
4. Linda sells insurance.
5. Linda is a bank teller and is active in the feminist movement.

A response to this question would be a conclusion. The reasons for such a conclusion would constitute the premises. For example, I could argue as follows:

Premise: Teachers are bright.
Premise: Teachers are college graduates.
Premise: A college degree isn’t needed to work in a bookstore.
Premise: Linda graduated from college.
Conclusion: Linda is more likely to be a teacher (than to work at a bookstore.)

In other words “Linda is a teacher” ranks higher than “Linda works in a bookstore and takes yoga classes.”

The following is another possible argument. (This is patterned what a group in our class did.)

Premise: There are five possible scenarios.
Premise: I wrote each on a sheet and randomly picked that Linda works in a bookstore.
Conclusion: Linda is more likely to work in a bookstore than anything else.

These are both examples of inductive arguments. This can be ascertained using the definitions, but perhaps an easier way to know that these are inductive arguments is to ask: How can I analyze the argument? An inductive argument can only be analyzed in terms of its strength. It requires personal judgment as to whether or not the conclusion follows from the premises. In examples above, do you think the arguments are sound?
On the other hand, a deductive argument can be analyzed in terms of its validity (whether or not the conclusion is direct result of the premises) and its soundness (in addition to being valid, the premises are true.) Contrast the previous two inductive arguments with the following deductive arguments:

**Premise:** All people who are bank tellers and active feminists are bank tellers.
**Premise:** Linda is a person.
**Conclusion:** It is more likely that Linda is a bank teller than that she is a bank teller AND active in the feminist movement.

**Premise:** All women who sell insurance are deeply concerned with issues of social justice.
**Premise:** Linda is deeply concerned with issues of social justice.
**Conclusion:** Linda sells insurance.

With each of these examples, we can analyze their validity and soundness. Given the premise “all bank tellers who are active in the feminist movement are bank tellers,” the conclusion is correct. Therefore, the argument is valid. Moreover, the premise is true, so we conclude that the argument is sound. The second argument is invalid, as can be verified by drawing a Venn Diagram. Thus, even though the conclusion may still be correct, the argument is not sound.

**Exercise 1.** Analyze the argument:

**Premise:** All women who are deeply concerned with issues of social justice sell insurance.
**Premise:** Linda is deeply concerned with issues of social justice.
**Conclusion:** Linda sells insurance.

**Exercise 2.** Come up with three inductive arguments and three deductive arguments related to what Linda does. For each of the inductive arguments analyze the strength of the argument. For each of the deductive arguments draw the accompanying Venn diagram and determine the validity and soundness (if possible) of the argument.

Mathematicians tend to be skeptical of inductive arguments. Even if the argument is very strong they are never satisfied until a deductive proof is given. Consider the following example.

**Conclusion:** If \( n \) is a natural number then the sum of the first \( n \) numbers is \( \frac{n(n+1)}{2} \).

(i.e. \( 1 + 2 + 3 + \cdots + (n - 1) + n = \frac{n(n+1)}{2} \).)

A possible inductive argument could be:

**Premise:** For \( n = 2 \) we have \( 1 + 2 = 3 = \frac{2(2+1)}{2} \).
**Premise:** For \( n = 3 \) we have \( 1 + 2 + 3 = 6 = \frac{3(3+1)}{2} \).
**Premise:** For \( n = 4 \) we have \( 1 + 2 + 3 + 4 = 10 = \frac{4(4+1)}{2} \).
**Premise:** I plugged it into my calculator for lots of other numbers and it always worked.

If you’re interested in seeing a deductive proof, I would be more than happy to show you one!