4.1:1 The limit should be 2, since $\frac{x^2 - 1}{x - 1}$ is equal to $x + 1$ for $x \neq 1$. Given $\epsilon > 0$, we pick $\delta = \epsilon$. Then, if $0 < |x - 1| < \delta$, we know that $x \neq 1$, so

$$\left| \frac{x^2 - 1}{x - 1} - 2 \right| = |(x + 1) - 2| = |x - 1| < \delta = \epsilon,$$

which proves that $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$.

4.1:2 Since $\frac{x^2 + x - 2}{x - 1}$ is continuous except at $x = 1$, it is continuous at $x = 2$, therefore, the limit as $x \to 2$ is equal to the value of the function at $x = 2$, i.e.,

$$\lim_{x \to 2} \frac{x^2 + x - 2}{x - 1} = \frac{2^2 + 2 - 2}{2 - 1} = 4.$$

4.1:3 First, we prove that the limit as $x \to 2$ of $\frac{x^2 - 4}{x - 2}$ is 4. Given $\epsilon > 0$, we pick $\delta = \epsilon$. Then, if $0 < |x - 2| < \delta$, we have that

$$\left| \frac{x^2 - 4}{x - 2} - 4 \right| = |(x + 2) - 4| = |x - 2| < \delta = \epsilon,$$

which proves the limit in question.

From example 4.1.13, if $\lim_{x \to a} g(x) = L$, then $\lim_{x \to a} g(x)^r = L^r$. Together, we deduce that

$$\lim_{x \to 2} \left( \frac{x^2 - 4}{x - 2} \right)^{3/2} = 4^{3/2} = 8.$$

4.2:1 Let $f(x) = \frac{1}{x}$. By definition,

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \to 0} \frac{x - (x + h)}{hx(x + h)} = \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}.$$

Or, alternatively,

$$f'(x) = \lim_{z \to x} \frac{\frac{1}{z} - \frac{1}{x}}{z - x} = \lim_{z \to x} \frac{x - z}{xz(z-x)} = \lim_{z \to x} \frac{-1}{xz} = -\frac{1}{x^2}.$$

4.2:2 Let $f(x) = x^2 + 3x$. By definition,

$$f'(x) = \lim_{h \to 0} \frac{(x + h)^2 + 3(x + h) - x^2 - 3x}{h} = \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} = \lim_{h \to 0} (2x + h + 3) = 2x + 3.$$

Or, alternatively,

$$f'(x) = \lim_{z \to x} \frac{z^2 + 3z - x^2 - 3x}{z - x} = \lim_{z \to x} \frac{(z - x)(z + x) + 3(z - x)}{z - x} = \lim_{z \to x} (z + x + 3) = 2x + 3.$$