1. For a set of nonzero vectors \( \{u, v, w\} \) in \( \mathbb{R}^n \), use words and/or math expressions to complete the following statements.

(a) A linear combination of \( u, v, \) and \( w \) is ...

any vector of the form \( ru + sv + tw \) for scalars \( r, s, t \).

(b) A vector \( b \in \mathbb{R}^n \) is in \( \text{span}\{u, v, w\} \) if ...

if \( b \) is a linear combination of \( u, v, w \), i.e., if we can find scalars \( r, s, t \) so that
\[
ru + sv + tw = b.
\]

(c) A vector \( b \in \mathbb{R}^n \) is not in \( \text{span}\{u, v, w\} \) if ...

if it is not a linear combination of \( u, v, w \), i.e., if \( ru + sv + tw = b \) is impossible no matter which scalars \( r, s, t \) are used.

(d) The set \( \{u, v, w\} \) is linearly dependent if ...

if 0 can be expressed as a nontrivial linear combination of \( u, v, w \), i.e., if we can find scalars \( r, s, t \) (not all zero) so that
\[
ru + sv + tw = 0.
\]

(e) The set \( \{u, v, w\} \) is linearly independent if ...

if 0 cannot be written as a nontrivial linear combination of \( u, v, w \), i.e., if the only way to make 0 is by taking \( 0u + 0v + 0w \).

2. (a) Give two vectors in \( \mathbb{R}^2 \) that are linearly dependent.

**Solution.** Any nonzero vector \( u \) and any multiple of \( u \) will be linearly dependent, for example
\[
u = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} -2 \\ 8 \end{bmatrix} = -2u.
\]

(b) Give two vectors in \( \mathbb{R}^2 \) that are linearly independent.

**Solution.** Any nonzero vectors \( u, v \) which are not multiples of each other will be independent, for example
\[
u = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}.
\]

(c) Give three vectors in \( \mathbb{R}^2 \) that are linearly dependent.

**Solution.** Any triple of vectors will work since the maximum number of independent vectors in \( \mathbb{R}^2 \) is two. Here is an example:
\[
u = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \quad v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \text{and} \quad w = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \quad \text{satisfy} \quad 7u + 6v - 5w = 0.
\]

(d) Give three vectors in \( \mathbb{R}^2 \) that are linearly independent.

**Solution.** This is impossible; the maximum number of linearly independent vectors in \( \mathbb{R}^n \) is \( n \).
(e) Give three vectors in $\mathbb{R}^3$ that are linearly dependent.

**Solution.** One way is to make the third vector a linear combination of the first two, for example

$$u = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \quad \text{and} \quad w = u + v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

(f) Give three vectors in $\mathbb{R}^3$ that are linearly independent.

**Solution.** If you pick 3 random vectors, chances are high that they are independent, but you would have to check. Here is a fairly simple but nontrivial example:

$$u = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad \text{and} \quad w = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}.$$

If we have $ru + sv + tw = 0$, then

$$\begin{bmatrix} 5r + 2s + 4t \\ 3s + 3t \\ 2t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The third component guarantees that $t = 0$; plugging in $t = 0$ to the second component, we see that $s = 0$; and plugging in $s = t = 0$ into the first component, we see that $r = 0$. Another way to see this is to make the augmented matrix

$$\begin{bmatrix} 5 & 2 & 4 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}.$$

Since there are 3 pivots and 3 vectors, there is a unique solution, namely the solution with coefficients all equal to 0.

3. Find all solutions (if any) to the following systems of equations.

(a) \[
\begin{align*}
2x - 3y &= -9 \\
-4x + 6y &= 18
\end{align*}
\]

**Solution.** The augmented system reduces to

$$\begin{bmatrix} 2 & -3 & -9 \\ -4 & 6 & 18 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & -3/2 & -9/2 \\ 0 & 0 & 0 \end{bmatrix},$$

so there are infinitely many solutions where $y$ is a free variable; if we pick $y = t$, the solution set is $x = -\frac{9}{2} + \frac{3}{2}t$, and $y = t$.

By the way, this line is the intersection of two lines, so both original lines are actually the same line and the solution is just a parametrization of the line.

(b) \[
\begin{align*}
x + 5y &= 7 \\
-2x + 2y &= 10
\end{align*}
\]

**Solution.** The augmented system eventually reduces to

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & 2 & 10 \end{bmatrix} \xrightarrow{\text{row operations}} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix},$$

so there is a unique solution $x = -3$ and $y = 2$. This is the intersection point of the two lines in $\mathbb{R}^2$. 
\[
\begin{align*}
  x - 3y &= -3 \\
  -x + y &= -1 \\
  2x - 5y &= -4
\end{align*}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
  1 & -3 & -3 \\
  -1 & 1 & -1 \\
  2 & -5 & -4
\end{bmatrix} \rightarrow
\begin{bmatrix}
  1 & 0 & 3 \\
  0 & 1 & 2 \\
  0 & 0 & 0
\end{bmatrix},
\]
so there is a unique solution \( x = 3 \) and \( y = 2 \). This is the intersection point of the three lines in \( \mathbb{R}^2 \).

(d) \[
\begin{align*}
  -2x - y + 3z &= 5 \\
  3x + 2y - 5z &= -2
\end{align*}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
  -2 & -1 & 3 & 5 \\
  3 & 2 & -5 & -2
\end{bmatrix} \rightarrow
\begin{bmatrix}
  1 & 0 & -1 & -8 \\
  0 & 1 & -1 & 11
\end{bmatrix},
\]
so there are infinitely many solutions. Since \( z \) is a free variable, we can set \( z = t \). The equation \( x - z = -8 \) becomes \( x = -8 + t \) and the equation \( y - z = 11 \) becomes \( y = 11 + t \). So, the solution set is
\[
x = -8 + t, \quad y = 11 + t, \quad \text{and} \quad z = t.
\]
This is the parametric equation of the line which is the intersection of the two planes in \( \mathbb{R}^3 \).

4. Find all solutions (if any) to the following linear combination problems. If infinitely many solutions exist, write down 3 of the linear combinations that work.

(a) Determine if \( \begin{bmatrix} 4 \\ 8 \end{bmatrix} \) is a linear combination of \( \begin{bmatrix} -1 \\ 4 \end{bmatrix} \) and \( \begin{bmatrix} 3 \\ -4 \end{bmatrix} \).

**Solution.** The equation we want to solve is
\[
x_1 \begin{bmatrix} -1 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}.
\]
The associated augmented system eventually reduces to
\[
\begin{bmatrix}
  -1 & 3 & 4 \\
  4 & -4 & 8
\end{bmatrix} \rightarrow
\begin{bmatrix}
  1 & 0 & 5 \\
  0 & 1 & 3
\end{bmatrix},
\]
so there is a unique solution \( x_1 = 5 \) and \( x_2 = 3 \). In other words, there is exactly one linear combination that works:
\[
5 \begin{bmatrix} -1 \\ 4 \end{bmatrix} + 3 \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}.
\]

(b) Determine if \( \begin{bmatrix} 5 \\ 6 \\ -12 \end{bmatrix} \) is a linear combination of \( \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \) and \( \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \).

**Solution.** The equation we want to solve is
\[
x_1 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -12 \end{bmatrix}.
\]
The associated augmented system eventually reduces to
\[
\begin{bmatrix}
2 & 3 & 5 \\
1 & -2 & 6 \\
-2 & 4 & -12
\end{bmatrix} \rightsquigarrow \begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{bmatrix},
\]
so there is a unique solution \(x_1 = 4\) and \(x_2 = -1\). In other words, there is exactly one linear combination that works:
\[
4 \begin{bmatrix}
2 \\
1 \\
-2
\end{bmatrix} - 3 \begin{bmatrix}
3 \\
-2 \\
4
\end{bmatrix} = \begin{bmatrix}
5 \\
6 \\
-12
\end{bmatrix}.
\]

(c) Determine if \(\begin{bmatrix}
1 \\
2 \\
4
\end{bmatrix}\) is a linear combination of \(\begin{bmatrix}
1 \\
0 \\
-1
\end{bmatrix}\) and \(\begin{bmatrix}
2 \\
-1 \\
3
\end{bmatrix}\).

**Solution.** The equation we want to solve is
\[
x_1 \begin{bmatrix}
1 \\
0 \\
-1
\end{bmatrix} + x_2 \begin{bmatrix}
2 \\
-1 \\
3
\end{bmatrix} = \begin{bmatrix}
1 \\
2 \\
4
\end{bmatrix}.
\]

The associated augmented system eventually reduces to
\[
\begin{bmatrix}
1 & 2 & 1 \\
0 & -1 & 2 \\
-1 & 3 & 4
\end{bmatrix} \rightsquigarrow \begin{bmatrix}
1 & 2 & 1 \\
0 & 1 & -2 \\
0 & 0 & 3
\end{bmatrix},
\]
so there is no solution (since the last equation says \(0x_1 + 0x_2 = 3\), which is impossible). In other words, there is no linear combination that works, so the vector on the right-hand side is not in the span of the vectors on the left.

(d) Determine if \(\begin{bmatrix}
-1 \\
13
\end{bmatrix}\) is a linear combination of \(\begin{bmatrix}
1 \\
3
\end{bmatrix}\), \(\begin{bmatrix}
-2 \\
2
\end{bmatrix}\), and \(\begin{bmatrix}
5 \\
-1
\end{bmatrix}\). The equation we want to solve is
\[
x_1 \begin{bmatrix}
1 \\
3
\end{bmatrix} + x_2 \begin{bmatrix}
-2 \\
2
\end{bmatrix} + x_3 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}.
\]

The associated augmented system eventually reduces to
\[
\begin{bmatrix}
1 & -2 & 5 \\
3 & 2 & -1 \\
1 & 0 & 1
\end{bmatrix} \rightsquigarrow \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -2 \\
0 & 0 & 2
\end{bmatrix},
\]
so there are infinitely many solutions. Since \(x_3\) is a free variable, we can set \(x_3 = t\). The equation \(x_1 + x_3 = 3\) becomes \(x_1 = 3 - t\) and the equation \(x_2 - 2x_3 = 2\) becomes \(x_2 = 2 + 2t\). For example, here are the linear combinations corresponding to \(t = -2, -1, 0, 1, 2, 3\).

\[
t = -2: \quad 5 \begin{bmatrix}
1 \\
3
\end{bmatrix} - 2 \begin{bmatrix}
-2 \\
2
\end{bmatrix} - 2 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
\[
t = -1: \quad 4 \begin{bmatrix}
1 \\
3
\end{bmatrix} + 0 \begin{bmatrix}
-2 \\
2
\end{bmatrix} - 1 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
\[
t = 0: \quad 3 \begin{bmatrix}
1 \\
3
\end{bmatrix} + 2 \begin{bmatrix}
-2 \\
2
\end{bmatrix} + 0 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
\[
t = 1: \quad 2 \begin{bmatrix}
1 \\
3
\end{bmatrix} + 4 \begin{bmatrix}
-2 \\
2
\end{bmatrix} + 1 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
\[
t = 2: \quad 1 \begin{bmatrix}
1 \\
3
\end{bmatrix} + 6 \begin{bmatrix}
-2 \\
2
\end{bmatrix} + 2 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
\[
t = 3: \quad 0 \begin{bmatrix}
1 \\
3
\end{bmatrix} + 8 \begin{bmatrix}
-2 \\
2
\end{bmatrix} + 3 \begin{bmatrix}
5 \\
-1
\end{bmatrix} = \begin{bmatrix}
-1 \\
13
\end{bmatrix}
\]
5. Find all solutions (if any) to the following matrix equations.

(a) \[
\begin{bmatrix}
2 & -4 & 10 \\
3 & 1 & 1 \\
-2 & 3 & -8 \\
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\
\end{bmatrix} =
\begin{bmatrix}
6 \\
5 \\
4 \\
\end{bmatrix}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
2 & -4 & 10 & 6 \\
3 & 1 & 1 & 5 \\
-2 & 3 & -8 & 4 \\
\end{bmatrix} \xrightarrow[]{}
\begin{bmatrix}
1 & -2 & 5 & 3 \\
0 & 1 & -2 & -2 \\
0 & 0 & 0 & 10 \\
\end{bmatrix},
\]
so there is no solution since the last row says that 0 = 10.

(b) \[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
0 & 1 & -1 & 2 \\
-1 & 0 & -1 & -1 \\
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\
\end{bmatrix} =
\begin{bmatrix}
1 \\
2 \\
3 \\
\end{bmatrix}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
1 & 2 & 0 & 4 & 1 \\
0 & 1 & -1 & 2 & 2 \\
-1 & 0 & -1 & -1 & 3 \\
\end{bmatrix} \xrightarrow[]{}
\begin{bmatrix}
1 & 0 & 0 & 2 & -3 \\
0 & 1 & 0 & 1 & 2 \\
0 & 0 & 1 & -1 & 0 \\
\end{bmatrix},
\]
so there are infinitely many solutions. We have \(x_4\) is a free variable, so we set \(x_4 = t\). The first equation \(x_1 + 2x_4 = -3\) becomes \(x_1 = -3 - 2t\); the second equation \(x_2 + x_4 = 2\) becomes \(x_2 = 2 - t\); the third equation \(x_3 - x_4 = 0\) becomes \(x_3 = t\). In other words, the solution set is
\[
x = \begin{bmatrix}
-3 - 2t \\
2 - t \\
t \\
\end{bmatrix} = \begin{bmatrix}
-3 \\
2 \\
0 \\
\end{bmatrix} + t \begin{bmatrix}
-2 \\
-1 \\
1 \\
\end{bmatrix}.
\]

You can check that
\[
\begin{bmatrix}
1 & 2 & 0 & 4 \\
0 & 1 & -1 & 2 \\
-1 & 0 & -1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
-3 - 2t \\
2 - t \\
t \\
\end{bmatrix} =
\begin{bmatrix}
1 \\
2 \\
3 \\
\end{bmatrix}.
\]

(c) \[
\begin{bmatrix}
-1 & 2 \\
1 & 3 \\
1 & -1 \\
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\
\end{bmatrix} =
\begin{bmatrix}
1 \\
4 \\
1 \\
\end{bmatrix}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
-1 & 2 & 1 \\
1 & 3 & 4 \\
1 & -1 & 1 \\
\end{bmatrix} \xrightarrow[]{}
\begin{bmatrix}
1 & -2 & -1 \\
0 & 1 & 1 \\
0 & 0 & 1 \\
\end{bmatrix},
\]
so there is no solution since the last row says that 0 = 1.

(d) \[
\begin{bmatrix}
-1 & 0 & 1 \\
1 & 1 & 2 \\
2 & 3 & 0 \\
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\
\end{bmatrix} =
\begin{bmatrix}
-3 \\
1 \\
7 \\
\end{bmatrix}
\]

**Solution.** The augmented system eventually reduces to
\[
\begin{bmatrix}
-1 & 0 & 1 & -3 \\
1 & 1 & 2 & 1 \\
2 & 3 & 0 & 7 \\
\end{bmatrix} \xrightarrow[]{}
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & -1 \\
\end{bmatrix},
\]
so there is a unique solution, namely

$$x = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$ 

6. Consider the matrix equation

$$\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 6 \end{bmatrix}.$$ 

(a) Show that the equation has a unique solution and find that solution. 

**Solution.** We reduce the augmented system,

$$\begin{bmatrix} 3 & -1 & 1 \\ 2 & 2 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix},$$

to see that the unique solution is

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$ 

(b) Write the corresponding system of equations, graph the two lines, and verify that your solution from (a) is the intersection point of the two lines.

**Solution.** The corresponding system of equation is

$$\begin{cases} 3x - y = 1 \\ 2x + 2y = 6 \end{cases}.$$ 

We graph these two lines and see that (1, 2) is the unique intersection point.

(c) Write the corresponding linear combination problem, and then use the graph of $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ and the column vectors of the matrix to verify that your solution from (a) gives the correct linear combination.

**Solution.** The corresponding linear combination problem and unique solution are

$$x_1 \begin{bmatrix} 3 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}.$$ 

We graph this linear combination below.
7. Consider the matrix equation \( \begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix} x = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \).

(a) Show that the system has no solution.

**Solution.** We reduce the augmented system,

\[
\begin{bmatrix} -2 & 1 & 2 \\ 6 & -3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix},
\]

to see that there is no solution since the last row says that 0 = 7.

(b) Graph the lines of the corresponding system of equations. How does this graph relate to the fact that there is no solution?

**Solution.** The corresponding system is

\[
\begin{align*}
-2x + y &= 2 \\
6x - 3y &= 1,
\end{align*}
\]

which are the two lines shown below. There is no solution exactly because the lines are parallel.

(c) Graph the vector \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \) along with the column vectors of the matrix. How can you interpret the fact that there is no solution in terms of linear combinations?

**Solution.** The column vectors lie in the same line, so every linear combination of them will lie on the line. Since \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \) is not on the line, it is impossible to get to it using the column vectors.
8. Consider following vectors in $\mathbb{R}^3$.

\[ a = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \quad c = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad d = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \quad e = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}. \]

For each of the sets below, determine if the set is linearly dependent or linearly independent. If the set is linearly dependent, give one explicit dependence relation between the vectors.

(a) $\{a, b\}$

**Solution.** They are linearly independent since $b$ is clearly not a multiple of $a$.

(b) $\{a, d\}$

**Solution.** They are linearly dependent since $d$ is a multiple of $a$. One dependence relation is $a + d = 0$.

(c) $\{a, b, c\}$

**Solution.** To check for independence, we reduce the matrix with $a, b, c$ as columns:

\[
\begin{bmatrix}
1 & 1 & 2 \\
2 & 3 & -3 \\
-1 & -2 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & -7 \\
0 & -1 & 3
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & -7 \\
0 & 0 & -4
\end{bmatrix}.
\]

Since there are 3 pivots and 3 vectors, $a, b, c$ are linearly independent.

(d) $\{a, b, e\}$

**Solution.** To check for independence, we reduce the matrix with $a, b, e$ as columns:

\[
\begin{bmatrix}
1 & 1 & 2 \\
2 & 3 & 5 \\
-1 & -2 & -3
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 1 \\
0 & -1 & -1
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
0 & 0 & 0
\end{bmatrix}.
\]

Since there are only 2 pivots, they are linearly dependent. To find a dependence relation, we note that the system augmented by the zero vector gives the equations

\[
x_1 + x_3 = 0 \quad \text{and} \quad x_2 + x_3 = 0.
\]

If we choose $x_3 = -1$, we get $x_1 = 1$ and $x_2 = 1$. Therefore, one dependence relation is

\[
a + b - e = 0, \quad \text{i.e.,} \quad \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
\]

In other words, we can think, for example, of $e$ as redundant because it can be expressed as $e = a + b$. 