1. (a) Use Sage to plot the lines $4x - 5y = 3$ and $-3x + 5y = 4$ and estimate their intersection point.

(b) Use Sage to reduce the corresponding augmented matrix $egin{bmatrix} 4 & -5 & | & 3 \\ -3 & 5 & | & 4 \end{bmatrix}$.

(c) Interpret (b) in terms of the following linear combination problem and use Sage to plot the vectors.

$$c_1 \begin{pmatrix} 4 \\ -3 \end{pmatrix} + c_2 \begin{pmatrix} -5 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$ 

2. (a) Use Sage to plot the three lines $3x + 2y = 1$, $x + 3y = 2$, and $-x + 5y = 4$ to see that there is no common intersection point.

(b) Verify this by using Sage to reduce the associated augmented system

$$\begin{bmatrix} 3 & 2 & | & 1 \\ 1 & 3 & | & 2 \\ -1 & 5 & | & 4 \end{bmatrix}$$

3. Consider the vectors $u = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$, $v = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$, and $w = \begin{pmatrix} 16 \\ -5 \\ -8 \end{pmatrix}$.

(a) Use Sage to reduce the augmented matrix $\begin{bmatrix} u & v & w & | & 0 \end{bmatrix}$.

(b) Use (a) to deduce that the set $\{u, v, w\}$ is linearly dependent and to find a dependence relation.

(c) Use Sage to plot the 3 vectors to verify that they lie in a common plane.

4. (a) Use Sage to show that the matrix equation

$$\begin{bmatrix} 5 & -2 & 16 \\ 2 & 3 & -5 \\ -1 & 2 & -8 \end{bmatrix} x = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$$

has no solution.

(b) Use Sage to plot the three planes in the corresponding system of equations, namely, $5x - 2y + 16z = 5$, $2x + 3y - 5z = 2$, and $x + 2y - 8z = 3$, in order to verify that they have no common intersection point.

5. (a) Use Sage to find the infinitely many solutions to

$$\begin{bmatrix} 5 & -2 & 16 \\ 2 & 3 & -5 \\ 1 & 2 & -8 \end{bmatrix} x = \begin{pmatrix} -7 \\ 20 \\ 13 \end{pmatrix}.$$ 

(b) Use Sage to plot the three planes in the corresponding system of equations, namely, $5x - 2y + 16z = -7$, $2x + 3y - 5z = 20$, and $x + 2y - 8z = 13$, in order to verify that their intersection is a line.