Consider the following shapes, all of whose vertices have integer coordinates.

1. For the following linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$, sketch the image of each of the shapes above.

   (a) The horizontal shear given by $P(x) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x$.

   (b) The rotation by $\pi/2$ given by $Q(x) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$.

   (c) The reflection about the line $y = -x$ given by $R(x) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} x$.

   (d) The dilation given by $S(x) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} x$.

   (e) The transformation given by $T(x) = \begin{bmatrix} -1 & 4 \\ 3 & 2 \end{bmatrix} x$.

2. Consider two transformations, a horizontal shear given by

   $$f(x) = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x$$

   and a vertical shear given by

   $$g(x) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x$$

   (a) The unit square is transformed under $f$ to a parallelogram. Find the coordinates of the parallelogram and sketch it.

   The first parallelogram is transformed under $g$ to a second parallelgram. Find the coordinates of the second parallelogram and sketch it.

   Find the matrix of the composition function $g \circ f$ by multiplying the matrices for $f$ and $g$ in the proper order. Verify that the columns of the resulting matrix are two of the vertices of second parallelogram.

   (b) Repeat part (a), but switch the order of composition: perform $g$ first and then $f$, i.e., $f \circ g$. Do you get the same thing?