INSTRUCTIONS: Complete the following problems on separate paper, using this page as a cover sheet. Justify your answers and show the relevant work. Each student needs to turn in their own assignment; however, you may work in groups, in which case you should list your collaborators below.

COLLABORATORS:

1. For each of the following matrix equations below, answer the following questions.
   
i) Use row reduction to find the solution set, i.e., the set of all possible $x$.
   
ii) Rewrite the matrix equation as a system of linear equations. Interpret the solution from (a) in terms of this system. Draw a sketch that illustrates the solution to the system.
   
iii) Rewrite the matrix system as an equation for solving a linear combination problem. Interpret the solution from (a) in terms of linear combinations. Draw a sketch that illustrates the linear combination.

(a) \[
\begin{bmatrix}
-2 & 3 \\
5 & -4
\end{bmatrix} x = \begin{bmatrix} 6 \\ 20 \end{bmatrix}
\]

(b) \[
\begin{bmatrix}
3 & 1 \\
2 & -1
\end{bmatrix} x = \begin{bmatrix} 10 \\ 5 \end{bmatrix}
\]

(c) \[
\begin{bmatrix}
-1 & 4 \\
-2 & 1
\end{bmatrix} x = \begin{bmatrix} 5 \\ -4 \end{bmatrix}
\]

(d) \[
\begin{bmatrix}
-1 & 3 \\
2 & -6
\end{bmatrix} x = \begin{bmatrix} -4 \\ 4 \end{bmatrix}
\]

(e) \[
\begin{bmatrix}
-5 & 10 \\
-2 & 4
\end{bmatrix} x = \begin{bmatrix} 30 \\ 12 \end{bmatrix}
\]

(f) \[
\begin{bmatrix}
1 & 1 \\
-1 & 2 \\
3 & 5
\end{bmatrix} x = \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}
\]

(g) \[
\begin{bmatrix}
1 & 1 \\
-1 & 2 \\
3 & 5
\end{bmatrix} x = \begin{bmatrix} 4 \\ 3 \\ -15 \end{bmatrix}
\]

2. Now consider an arbitrary $2 \times 2$ matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(a) Under what conditions on $A$ will the matrix equation $Ax = y$ always have a unique solution? [Hint: think of interpreting the matrix equation as a system of equations or a linear combination problem.]

(b) Under what conditions on $A$ and $y$ will the matrix equation have no solution?

(c) Under what conditions on $A$ and $y$ will the matrix equation have infinitely many solutions?