**Tangent Planes.** The tangent plane to a function \( f(x, y) \) at the point \( a = (x_0, y_0) \) is
\[
z = f(a) + \nabla f(a) \cdot (x - x_0, y - y_0);
\]
if \( \nabla f(a) = (A, B) \), then the normal vector to the tangent plane is \( \mathbf{n} = (A, B, -1) \).

**Chain Rule.** If \( f \) is a function of \((x, y, z)\) and each of \(x, y, z\) is a function of \((s, t)\), then
\[
\frac{df}{ds} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial s}
\]
and
\[
\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}
\]

**Local Extrema.** A critical point of a function \( f(x, y) \) occurs when \( \nabla f = (0, 0) \). Compute \( D = f_{xx} f_{yy} - f_{xy}^2 \).
- If \( D < 0 \), then the critical point is a saddle point.
- If \( D > 0 \) and \( f_{xx} < 0 \), then the critical point is a local maximum.
- If \( D > 0 \) and \( f_{xx} > 0 \), then the critical point is a local minimum.
- If \( D = 0 \), then the analysis is inconclusive.

**Lagrange Multipliers.** The extrema of a function \( f \) subject to a constraint function \( g = 0 \) occurs when
\[
\nabla f = \lambda \nabla g.
\]

**Double Integrals.** The integral of a function \( f(x, y) \) over a region \( R \) in the \( xy \)-plane is
\[
\iint_R f(x, y) \, dA.
\]
In Cartesian coordinates, \( dA = dx \, dy \). In polar coordinates, \( dA = r \, dr \, d\theta \).

**Triple Integrals.** The integral of a function \( f(x, y, z) \) over a solid \( R \) in \( \mathbb{R}^3 \) is
\[
\iiint_R f(x, y, z) \, dV.
\]
In Cartesian coordinates, \( dV = dx \, dy \, dz \). In cylindrical coordinates, \( dV = r \, dr \, d\theta \, dz \). In spherical coordinates, \( dV = \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \).

**Cartesian/cylindrical/spherical coordinates.** The equations relating the various coordinate systems:
\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta \\
y &= x \tan \theta \\
z &= \rho \cos \phi \\
r &= \rho \sin \phi
\end{align*}
\]

**Change of Variables.** Suppose that \( f \) is a function of \((x, y)\) and that \( x \) and \( y \) are functions of \((u, v)\), i.e., that \((x, y) = g(u, v)\) for a transformation \( g : \mathbb{R}^2 \to \mathbb{R}^2 \). The Jacobian of the change of variables is
\[
J(u, v) = \begin{bmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{bmatrix}.
\]
For a region \( R \) in the \( xy \)-plane, and its corresponding region \( S = g^{-1}(R) \) in the \( uv \)-plane,
\[
\iint_R f(x, y) \, dx \, dy = \iint_S f(x(u, v), y(u, v)) |J(u, v)| \, du \, dv.
\]

**Mass and Center of Mass.** If \( R \) is a region in the \( xy \)-plane with a density \( \delta(x, y) \), the mass of the region is
\[
m = \iint_R \delta(x, y) \, dA,
\]
and the center of mass is given by
\[
\bar{x} = \frac{1}{m} \iint_R x \delta(x, y) \, dA \quad \text{and} \quad \bar{y} = \frac{1}{m} \iint_R y \delta(x, y) \, dA.
\]