1. For each function, find all of its critical points and then classify each point as a local extremum or saddle point.
   (a) \( f(x, y) = 2x^3 + 6xy + 3y^2 \)
   (b) \( g(x, y) = x(y^2 - 4)e^x \)

2. Consider the surface given by \( z = x + 2y - y^2 \).
   (a) Find the gradient of the surface at the point \((4, -1, 1)\).
   (b) Sketch the level curves of the surface corresponding to \( z = -2, -1, 0, 1, 2 \). Then sketch the gradient vector from part (a), emanating from the point \((4, -1)\) on the level curve \( z = 1 \).
   (c) Find the equation of the plane tangent to the surface at the point \((4, -1, 1)\).

3. Let \( a, b \), and \( P \) be constants. Use the method of Lagrange multipliers to show that the function \( f(x, y) = xy \) subject to the constraint \( ax + by = P \) has a maximum value of \( \frac{P^2}{4ab} \).

4. Let \( w = x^2y + 4xz \).
   (a) Find the gradient of \( w \) as a function of \((x, y, z)\).
   (b) If \( x = s^2t, y = st^2 \), and \( z = s + 2t \), use the chain rule to find the gradient of \( w \) as a function of \((s, t)\).

5. Compute the integral \( \iint_S (25 - x^2 - y^2) \, dA \) for each of the regions pictured below.

6. Consider the triangle \( T \) with vertices \((0, 0)\), \((3, 3)\), and \((2, -1)\) and with density function \( \delta(x, y) = 2 \).
   (a) Find the mass and center of mass of the triangle directly.
   (b) Find the mass and center of mass by performing the change of variables
       \[ x = 2u + v \quad \text{and} \quad y = -u + v. \]

7. Find the volumes of the following solids.
   (a) The first octant solid bounded by the coordinate planes and the planes \( y = 2 \) and \( x + 2y + 3z = 6 \).
   (b) The first octant solid bounded by the \( xz \)-plane, the \( xy \)-plane, the plane \( y = x \), and in between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 4 \).
   (c) The solid bounded by the \( xy \)-plane, the plane \( y + z = 4 \), and the cylinder \( x^2 + y^2 = 4 \).