1. Consider the surface given by $z = y - x^2 + 2$.

   (a) Find the equations of the level curves corresponding to $z = 0, 1, 2, 3, 4$. Then plot all of them on the graph provided to make a contour map.

   $z = 0$: $y = x^2 - 2$
   $z = 1$: $y = x^2 - 1$
   $z = 2$: $y = x^2$
   $z = 3$: $y = x^2 + 1$
   $z = 4$: $y = x^2 + 2$

   (b) Find the gradient to the curve at the point $(1, 2, 3)$. Plot the gradient on the contour map above, emanating from the point $(1, 2)$ on the level curve $z = 3$.

   **Solution.** The gradient is $\nabla z = (-2x, 1)$. At the point $(1, 2, 3)$, the gradient is $\nabla z = (-2, 1)$. This is the direction of greatest incline.

2. Consider the surface given by the equation $f(x, y) = x^2y - xy + ye^x$.

   (a) Find the gradient of $f$ at the point $a = (0, 3)$.

   **Solution.** The gradient of $f$ is $\nabla f = (2xy - y + ye^x, x^2 - x + e^x)$. At the point $a = (0, 3)$, the gradient is $\nabla f = (0, 1)$.

   (b) Use the gradient to find the equation of the tangent plane to $f$ at the point $a = (0, 3)$.

   **Solution.** The equation of the tangent plane to $f$ at $a$ is
   
   $z = f(a) + \nabla f \cdot ((x, y)) - a$,
   
   so the tangent plane at $(0, 3)$ is
   
   $z = f(0, 3) + (0, 1) \cdot (x - 0, y - 1) = 3 + 0(x - 0) + 1(y - 1) \rightarrow z = y + 2$. 