13.1.4 We break the original region 
\[ R = \{(x, y) : 1 \leq x \leq 4, \ 0 \leq y \leq 2 \} \]
into the subsections given in the piecewise definition of \( f(x, y) \):
\[ R_1 = \{(x, y) : 1 \leq x \leq 4, \ 0 \leq y < 1 \}, \]
\[ R_2 = \{(x, y) : 1 \leq x < 3, \ 1 \leq y \leq 2 \}, \]
\[ R_3 = \{(x, y) : 3 \leq x < 4, \ 1 \leq y \leq 2 \}. \]

Since \( f(x, y) = 2 \) on \( R_1 \), \( f(x, y) = 3 \) on \( R_2 \), and \( f(x, y) = 1 \) on \( R_3 \), then
\[
\int \int_R f(x, y) \, dA = \int \int_{R_1} f(x, y) \, dA + \int \int_{R_2} f(x, y) \, dA + \int \int_{R_3} f(x, y) \, dA
\]
\[ = 2 \text{Area}(R_1) + 3 \text{Area}(R_2) + \text{Area}(R_3)
\]
\[ = 13. \]

13.1.12 The region \( R \) is partitioned into 6 equal squares \( R_1, \ldots, R_6 \), with center points \( (x_1, y_1), \ldots, (x_6, y_6) \), as pictured below.

The function value of \( f(x, y) = \frac{1}{8} (48 - 4x - 3y) \) at each of the center points is given by
\[
\begin{array}{c|c|c|c|c|c|c}
(x_k, y_k) & (1, 1) & (3, 1) & (5, 1) & (1, 3) & (3, 3) & (5, 3) \\
f(x_k, y_k) & 41/6 & 33/6 & 25/6 & 27/6 & 19/6 & \end{array}
\]

Therefore, the integral is approximated by the Riemann sum as follows:
\[
\int \int_R \frac{1}{8} (48 - 4x - 3y) \, dA \approx \sum_{k=1}^{6} f(x_k, y_k) \Delta A_k
\]
\[ = \frac{41}{6} (4) + \frac{33}{6} (4) + \frac{25}{6} (4) + \frac{35}{6} (4) + \frac{27}{6} (4) + \frac{19}{6} (4)
\]
\[ = 120. \]
13.2.6
\[ \int_{-1}^{1} \int_{-1}^{1} (x^2 + y^2) \, dx \, dy = \int_{-1}^{1} \left( \frac{x^3}{3} + xy^2 \right|_{-1}^{1} \, dy = \int_{-1}^{1} \left( \frac{7}{3} + y^2 \right) \, dy = \frac{7y}{3} + \frac{y^3}{3} \bigg|_{-1}^{1} = \frac{14}{3} + \frac{2}{3} = \frac{16}{3} \].

13.2.10
\[ \int_{0}^{1} \int_{0}^{1} xe^{xy} \, dy \, dx = \int_{0}^{1} \left( e^{xy} \right|_{0}^{1} \, dx = e^{x} - x \bigg|_{0}^{1} = (e - 1) - (0 - 0) = e - 2. \]

13.2.12
\[ \int_{0}^{1} \int_{0}^{1} \frac{y}{(xy + 1)^2} \, dx \, dy = \int_{0}^{1} \left( \frac{-1}{xy + 1} \right|_{0}^{1} \, dy = \int_{0}^{1} \left( \frac{-1}{\sqrt{y} + 1} + 1 \right) \, dy = -\ln(y + 1) + y \bigg|_{0}^{1} = -\ln 2 + 1. \]

13.2.20
\[ \iint_{R} xy\sqrt{1 + x^2} \, dA = \int_{1}^{\sqrt{3}} \int_{0}^{1} xy\sqrt{1 + x^2} \, dx \, dy = \int_{1}^{\sqrt{3}} \left( \frac{y}{3}(1 + x^2)^{3/2} \right|_{0}^{\sqrt{3}} \, dy = \int_{1}^{\sqrt{3}} \frac{7y^2}{3} \, dy = \frac{7y^3}{6} \bigg|_{1}^{\sqrt{3}} = \frac{7}{2}. \]

13.2.22
\[ V = \int_{0}^{2} \int_{0}^{3} (25 - x^2 - y^2) \, dy \, dx = \int_{0}^{2} \left( 25y - x^2y - \frac{y^3}{3} \right|_{0}^{3} \, dx = \int_{0}^{2} \left( 66 - 3x^2 \right) \, dx = 66x - x^3 \bigg|_{0}^{2} = 124. \]