12.1.18 The level curve at \( z = k \) is the line through the origin of slope \( k \); if \( z = k \), then \( y = kx \).

12.1.24 For a fixed value \( V = k \), the equipotential curve is

\[
\sqrt{(x-2)^2 + (y+3)^2} = 4k,
\]

which is the upper half of the circle of radius \( 4k \) centered at \( (2, -3) \). For \( k = 1/2, 1, 2, 4 \), we get the circles or radius 2, 4, 8, and 16.

12.1.34 The level surfaces for the function are obtained by setting \( f(x, y, z) = k \), which gives

\[ k = 100x^2 + 16y^2 + 25z^2. \]

Dividing both sides by \( k \), we get equations of ellipsoids of various sizes.

12.2.10 The partial derivatives are

\[
\frac{\partial f}{\partial s} = \frac{2s}{s^2 - t^2} \quad \text{and} \quad \frac{\partial f}{\partial t} = \frac{-2t}{s^2 - t^2}.
\]
12.2.18 The first partial derivatives are \( \frac{\partial f}{\partial x} = 15x^2(x^3 + y^2)^4 \) and \( \frac{\partial f}{\partial y} = 10y(x^3 + y^2)^4 \). The mixed partials should be equal, which we verify by taking the partial of the first with respect to \( y \) and the partial of the second with respect to \( x \):

\[
\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left[ 15x^2(x^3 + y^2)^4 \right] = 120x^2y(x^3 + y^2)^3
\]

\[
\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left[ 10y(x^3 + y^2)^4 \right] = 120x^2y(x^3 + y^2)^3.
\]

12.2.34 The first partial derivatives are \( \frac{\partial f}{\partial x} = 8x^4x^2 + 4y^2 \) and \( \frac{\partial f}{\partial y} = 8y^4x^2 + 4y^2 \). The second partial derivatives are

\[
\frac{\partial^2 f}{\partial x^2} = \frac{(4x^2 + 4y^2)(8) - 8x(8x)}{(4x^2 + 4y^2)^2} = -32x^2 + 32y^2
\]

and

\[
\frac{\partial^2 f}{\partial y^2} = \frac{(4x^2 + 4y^2)(8) - 8y(8y)}{(4x^2 + 4y^2)^2} = 32x^2 - 32y^2
\]

which means that \( f \) is harmonic since

\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.
\]

12.2.40 (a) \( f_x(x, y, z) = 12x^2(x^3 + y^2 + z)^3 \).

(b) \( f_y(0, 1, 1) = 8y(x^3 + y^2 + z)^3 \bigg|_{(0,1,1)} = 8(0 + 1 + 1)^3 = 64 \)

(c) \( f_{zz}(x, y, z) = \frac{\partial}{\partial z} f_z = \frac{\partial}{\partial z} [4(x^3 + y^2 + z)^3] = 12(x^3 + y^2 + z)^2. \)

12.3.20 The function is defined and continuous as long as the denominator is defined and nonzero. The denominator is defined and nonzero if and only if \( 1 + x + y > 0 \). This set is the half-plane \( y > -x - 1 \) as pictured below.

![Graph of a plane]

12.4.4 The gradient of \( f \) is \( \nabla f = (2xy \cos y, -2x^2 \sin y + x^2 \cos y) \).

12.4.14 The gradient of \( f \) is \( \nabla f = (2x/y, -x^2/y^2) \); evaluated at \( (2, -1) \), we get \( \nabla f = (-4, -4) \). Since \( f(2, -1) = 2^2/(-1) = -4 \), the equation of the tangent plane is \( z = -4 - 4(x - 2) - 4(y + 1) \).
12.5.4 The gradient of $f$ is $\nabla f = \langle 2x - 3y, -3x + 4y \rangle$; evaluated at $(-1, 2)$, we get $\nabla f = \langle -8, 11 \rangle$. The directional derivative in the direction $a = \langle 2, -1 \rangle$ is
\[
\frac{\nabla f \cdot a}{||a||} = \frac{-27}{\sqrt{5}}.
\]

12.5.14 The gradient of $f$ is $\nabla f = \langle 3\cos(3x - y), -\cos(3x - y) \rangle$; evaluated at $(\pi/6, \pi/4)$, we get
\[
\nabla f = \left\langle \frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle = \frac{1}{\sqrt{2}}(3, -1).
\]

Therefore, the function decreases most rapidly in the direction $\langle -3, 1 \rangle$, opposite to $\nabla f$. 