11.6.2 Taking the point $P_0 = \langle 2, -1, -5 \rangle$ and the direction vector $v = \langle 5, -1, 8 \rangle$, we get that the parametric line is given by

$$L(t) = P_0 + tv = \langle 2 + 5t, -1 - t, -5 + 8t \rangle.$$  

Notice that $L(1) = \langle 7, -2, 3 \rangle$ is the other point.

11.6.24 The curve is $r(t) = \langle 2t^2, 4t, t^3 \rangle$, so the derivative is $r'(t) = \langle 4t, 4, 3t^2 \rangle$. Plugging in $t = 1$, we get that the point on the curve is $r(1) = \langle 2, 4, 1 \rangle$ and that the tangent vector is $r'(1) = \langle 4, 4, 3 \rangle$. Therefore, the equation of the tangent line is

$$L(t) = \langle 2 + 4t, 4 + 4t, 1 + 3t \rangle.$$
11.6.28 (a) The curve lies on a sphere centered at the origin if and only if it is always the same distance from the origin. We compute the length of the vector \( \mathbf{r}(t) = \left< \sin t \cos t, \sin^2 t, \cos t \right> \) at an arbitrary value \( t \):

\[
\|\mathbf{r}(t)\| = \sqrt{\sin^2 t \cos^2 t + \sin^4 t + \cos^2 t} = \sqrt{\sin^2 t(\cos^2 t + \sin^2 t)} + \cos^2 t = 1.
\]

Since it is always length 1, the curve lies on the sphere of radius 1 centered at the origin.

(b) The derivative is

\[
\mathbf{r}'(t) = \left< -\sin^2 t + \cos^2 t, 2 \sin t \cos t, -\sin t \right> = \left< \cos 2t, \sin 2t, -\sin t \right>.
\]

At time \( t = \pi/6 \), the point on the curve is

\[
\mathbf{P}_0 = \mathbf{r}(\pi/6) = \left< \frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2} \right>
\]

and the tangent vector is

\[
\mathbf{v} = \mathbf{r}'(\pi/6) = \left< \frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{1}{2} \right>.
\]

Therefore, the equation of the tangent line is

\[
\mathbf{L}(t) = \mathbf{P}_0 + t\mathbf{v} = \left< \frac{1}{4}(\sqrt{3} + 2t), \frac{1}{4}(1 + 2t\sqrt{3}), \frac{1}{4}(\sqrt{3} - t) \right>.
\]

The tangent line crosses the \( xy \)-plane when \( z = 0 \), i.e., when \( t = \sqrt{3} \); that is, the tangent crosses at the point

\[
\mathbf{Q}_0 = \mathbf{L}(\sqrt{3}) = \left< \frac{3\sqrt{3}}{4}, \frac{7}{4}, 0 \right>.
\]
11.7.4 The curve is \( \mathbf{r}(t) = (5 \cos t, 2t, 5 \sin t) \). The velocity function is 
\[
\mathbf{v}(t) = (-5 \sin t, 2, 5 \cos t),
\]
so the tangent vector at \( t = \pi \) is 
\[
\mathbf{v} = \mathbf{v}(\pi) = (0, 2, -5).
\]
The unit tangent function is 
\[
\mathbf{T}(t) = \frac{\mathbf{v}(t)}{||\mathbf{v}(t)||} = \frac{1}{\sqrt{29}} (-5 \sin t, 2, 5 \cos t)
\]
and the unit tangent vector at \( t = \pi \) is 
\[
\mathbf{T} = \mathbf{T}(\pi) = \frac{1}{\sqrt{29}} (0, 2, -5).
\]
The acceleration function is 
\[
\mathbf{a}(t) = (-5 \cos t, 0, -5 \sin t)
\]
so the accleration vector at \( t = \pi \) is 
\[
\mathbf{a} = \mathbf{a}(\pi) = (5, 0, 0).
\]
The derivative of the unit tangent function is 
\[
\mathbf{T}'(t) = \frac{1}{\sqrt{29}} (-5 \cos t, 0, -5 \sin t)
\]
so the curvature at \( t = \pi \) is 
\[
\kappa = \frac{||\mathbf{T}'(\pi)||}{||\mathbf{v}(\pi)||} = \frac{5}{29}.
\]
11.7.8 The curve is $\mathbf{r}(t) = \langle t^3/3, t^2/2 \rangle$, so the derivative and second derivative are

$$\mathbf{r}'(t) = \langle t^2, t \rangle \quad \text{and} \quad \mathbf{r}''(t) = \langle 2t, 1 \rangle.$$ 

The curvature is given by the formula

$$\kappa(t) = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}} = \frac{|t^2 - 2t^2|}{[t^4 + t^2]^3/2} = \frac{t^2}{t^3[t^2 + 1]^{3/2}} = \frac{1}{t(t^2 + 1)^{3/2}}.$$ 

At time $t = 1$, the curvature is $\kappa = 2^{-3/2}$. This means the circle of tangency has radius of $2\sqrt{2}$, as pictured below.

11.7.28 The function and its two derivatives are

$$\mathbf{r}(t) = \langle \sin 3t, \cos 3t, t \rangle$$

$$\mathbf{r}'(t) = \langle 3 \cos 3t, -3 \sin 3t, 1 \rangle$$

$$\mathbf{r}''(t) = \langle -9 \sin 3t, -9 \cos 3t, 0 \rangle$$

At $t = \pi/9$, we have

$$\mathbf{r} = \frac{1}{18} \langle 9\sqrt{3}, 9, 2\pi \rangle, \quad \mathbf{r}' = \frac{1}{2} \langle 3, -3\sqrt{3}, 2 \rangle, \quad \text{and} \quad \mathbf{r}'' = -\frac{9}{2} \langle \sqrt{3}, 1, 0 \rangle.$$ 

Then

$$\mathbf{r}' \cdot \mathbf{r}'' = \frac{9}{4} (3\sqrt{3} - 3\sqrt{3} + 0) = 0$$

and

$$\mathbf{r}' \times \mathbf{r}'' = -\frac{9}{4} \det \begin{bmatrix} i & j & k \\ 3 & -3\sqrt{3} & 2 \\ \sqrt{3} & 1 & 0 \end{bmatrix} = -\frac{9}{4} \langle -2, 2\sqrt{3}, 12 \rangle = -\frac{9}{2} \langle -1, \sqrt{3}, 6 \rangle,$$

so

$$||\mathbf{r}'|| = \frac{1}{2} \sqrt{9 + 27 + 4} = \sqrt{10}$$

and

$$||\mathbf{r}' \times \mathbf{r}''|| = \frac{9}{2} \sqrt{1 + 3 + 36} = 9\sqrt{10}.$$
From these, we get all the information we need:

\[
\mathbf{T} = \frac{\mathbf{r}'}{||\mathbf{r}'||} = \frac{1}{2\sqrt{10}} \langle 3, -3\sqrt{3}, 2 \rangle \\
\alpha_T = \frac{\mathbf{r}' \cdot \mathbf{r}''}{||\mathbf{r}'||} = 0 \\
\alpha_N = \frac{||\mathbf{r}' \times \mathbf{r}''||}{||\mathbf{r}'||} = 9 \\
\mathbf{N} = \frac{\mathbf{r}'' - \alpha_T \mathbf{T}}{\alpha_N} = -\frac{1}{2} \langle \sqrt{3}, 1, 0 \rangle \\
\kappa = \frac{||\mathbf{r}' \times \mathbf{r}''||}{||\mathbf{r}'||^3} = \frac{9}{10} \\
\mathbf{B} = \mathbf{T} \times \mathbf{N} = -\frac{1}{2\sqrt{10}} \langle -1, \sqrt{3}, 6 \rangle.
\]

11.7.40 The curvature of \( y = \ln(\cos x) \) for \( x \) in \((-\pi/2, \pi/2)\) is given by

\[
\kappa = \frac{|y''|}{\left(1 + (y')^2\right)^{3/2}} = \frac{\sec^2 x}{(1 + \tan^2 x)^{3/2}} = \frac{\sec^2 x}{\sec^3 x} = \cos x.
\]

The maximum curvature occurs at \( x = 0 \), which is the point \((0, 0)\) where \( \kappa = \cos x \) takes its maximum value of \( \kappa = 1 \). Note that this is where the circle of tangency has its smallest radius of \( r = 1 \).