MATH 1210-6
Spring 2003
Midterm exam III

Student Name: _____________________________

Student ID Number: ___________________________

Course Abbreviation and Number: Math 1210
Course Title: Calculus I
Instructor: Vladimir Vinogradov

Date of Exam: April 7, 2003
Time Period: Start time: 12:55 pm   End Time: 1:55 pm
Duration of Exam: 1 hours
Number of Exam Pages: 6
   (including this cover sheet)
Exam Type: Closed Book
Additional Materials Allowed: Calculator

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>VALUE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

*) The bonus question counts for 10 points maximum.
1. (20 points) Find the indefinite integral of:

\[ \int 24x(x^2 - 1)^3 \, dx \]

ANSWER: _______________________

2. (20 points) Find the function whose value at 1 is 0 and whose derivative is given:

\[ \frac{dy}{dx} = \frac{3(x^2 + 1)}{(x^3 + 3x)^2} \]

ANSWER: _______________________

2
3. (20 points) Calculate the definite integrals:

\[
\int_{\pi/6}^{\pi/3} 4\left(\sin(4x) - \cos(4x)\right) \, dx
\]

**ANSWER:**

4. (20 points) Find the solution to the following differential equation such that \( y(0) = 1 \)

\[
\frac{dy}{dx} = \sqrt{x}y^2 + 2x^2y^2
\]

**ANSWER:**
5. (20 points) What is the area of the region bounded by the curves $y_1 = 5 - (x - 2)^2$ and $y_2 = 1$.

ANSWER: ______________________
Bonus question (10 points). Evaluate
\[
\frac{d}{dx} \int_{-x^2}^{x^2} f(t) dt
\]

\text{ANSWER: } \underline{5}
Useful formulae

Differentiation

Product rule: 
(\( f(x)g(x) \))' = \( f'(x)g(x) + f(x)g'(x) \)

Quotient rule: 
(\( \frac{f(x)}{g(x)} \))' = \( \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \)

Chain rule: \( \frac{d}{dx} f(g(x)) = \frac{df}{dg} \cdot \frac{dg}{dx} \)

Power rule \( (x^\alpha)' = \alpha x^{\alpha-1} \)

Trigonometric functions: 
\( (\cos x)' = -\sin x \), \( (\sin x)' = \cos x \)

Integration

\[ \int f(g(x))g'(x)dx = \int f(u)du = F(g(x)) + C \]
\[ \int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du = F(g(b)) - F(g(a)) \]

\[ \cos(0) = 1, \ \sin(0) = 0 \]
\[ \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \ \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \]
\[ \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \ \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \]
\[ \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \ \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \]
\[ \cos\left(\frac{\pi}{2}\right) = 0, \ \sin\left(\frac{\pi}{2}\right) = 1 \]
\[ \cos(\pi) = -1, \ \sin(\pi) = 0 \]