Math 3070 § 1.
Treibergs $\sigma\pi$

Solution to devore 4.2.26

Name: Solutions
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(26.) *let X be the total medical expenses (in \$1000's) incurred by a particular individual during a given year.Suppose that the distribution is well approximated by a continuous one with pdf*

$$f(x) = \begin{cases} \dfrac{k}{\left(1+\frac{x}{2.5}\right)^7}, & \text{if } x \ge 0; \\ 0 & \text{otherwise.} \end{cases}$$

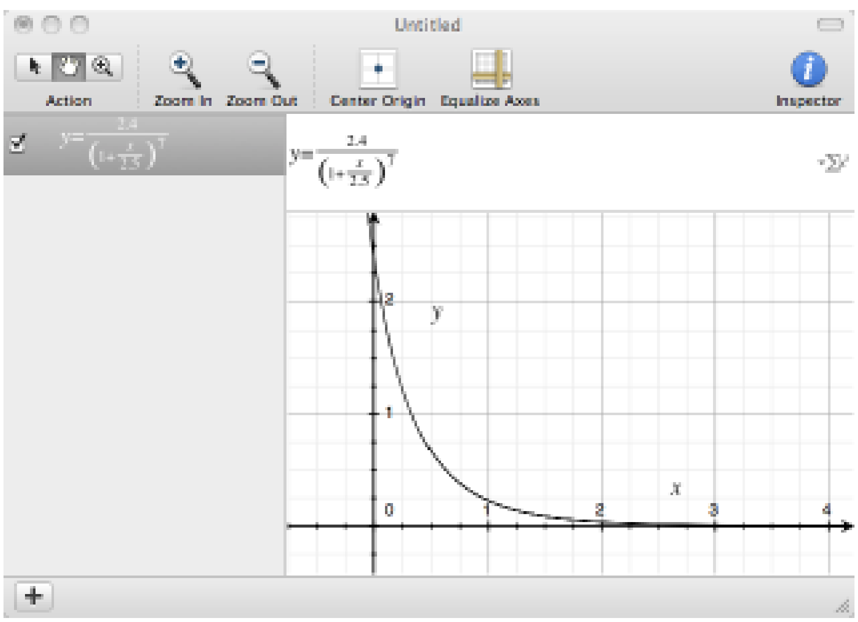
(a.) *What is the value ofk?*
(b.) *Graph the pdf of X.*
(c.) *What is the expected value and standard deviation of the medical expenses?*
(d) *The individual is covered by an insurance plan that entails a deductible of \$ 500 provision (so the first \$ 500 of expenses are paid by the individual.) then the plan will pay 80% of any additional expenses exceeding \$ 500, and the maximum payment by the individual (including the deductible amount) is \$ 2500. let Y denote the the amount of this individual's medical expenses paid by the insurance company. What is the expectede value of Y?*

(a.) By the total probability formula, substituting $u = 1 + \frac{x}{2.5}$ yields

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} k\left(1+\frac{x}{2.5}\right)^{-7} dx = k\int_1^{\infty} 2.5u^{-7}\,du = 2.5k\left[\frac{u^{-6}}{-6}\right]_1^{\infty} = \frac{2.5}{6}k$$

so that $k = \frac{12}{5}$.

(b.)



(c.) Compute $E(X)$ and $E(X^2)$:

$$E(X) = \int_0^{\infty} kx\left(1+\frac{x}{2.5}\right)^{-7} dx = k\int_1^{\infty} 2.5(2.5u-2.5)u^{-7}\,du = \frac{2.5^2k}{30} = \frac{1}{2}$$

$$E(X^2) = \int_0^{\infty} kx^2\left(1+\frac{x}{2.5}\right)^{-7} dx = k\int_1^{\infty} 2.5(2.5u-2,5)^2u^{-7}\,du = \frac{2.5^3k}{60} = \frac{5}{8};$$

so $\mu = E(X) = \frac{1}{2}$ and $\sigma^2 = E(X^2) - E(X)^2 = \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$ so $\sigma = \sqrt{\frac{3}{8}}$.

(d.) Follow the hint in the book. The payout $h(x)$ is zero if $x \le 500$, the deductible amount. Thereafter it pays .8 of the cost over 500, namely $.8(x - 500)$ until the customer has paid \$ 2500. If z is the amount where customer has paid exactly \$ 2500, then

$$2500 = x - \frac{4}{5}(x - 500)$$

which means $z = 10500$. Thereafter the company pays the full difference. Thus

$$y = h(x) = \begin{cases} 0, & \text{if } x \le 500; \\ \frac{4}{5}(x - 500), & \text{if } 500 \le x \le 1050; \\ x - 2500, & \text{if } 1050 \le x. \end{cases}$$

Then the expected payout is

$$\begin{aligned} E(h(X)) &= \int_0^\infty h(x) k \left(1 + \frac{x}{2.5}\right)^{-7} dx \\ &= k \int_{500}^{10500} \frac{4}{5}(x - 500)\left(1 + \frac{x}{2.5}\right)^{-7} dx + k \int_{10500}^{\infty} (x - 2500)\left(1 + \frac{x}{2.5}\right)^{-7} dx \\ &= \frac{4k}{5} \int_{201}^{4201} (2.5u - 502.5) u^{-7}\, du + k \int_{4201}^{\infty} ((2.5u - 2502.5) u^{-7}\, du \\ &= \frac{12}{5}\left\{\frac{2}{5}\left(\frac{1}{201^5} - \frac{1}{4201^5}\right) - \frac{402}{6}\left(\frac{1}{201^6} - \frac{1}{4201^6}\right)\right\} + \frac{12}{5}\left\{\frac{1}{2 \cdot 4201^5} - \frac{2502.5}{6 \cdot 4201^6}\right\}. \end{aligned}$$