Math 3070 § 1. Treibergs

Probability Plots for Normal, Exponential and Weibull Variables

Name: Example

October 7, 2010

Data File Used in this Analysis:

```
# M3070 - 1             Geyser Data            Oct. 7, 2010
# Data from Navidi, "Principles of Statistics for Engineers and Scientists"
# McGraw Hill, 2010
# The following are durations in minutes of 40 consecutive time intervals
# between eruptions of the Old Faithful geyser in Yellowstone National Park
#
Dormant
91
51
79
53
82
51
76
82
84
53
86
51
85
45
88
51
80
49
82
75
73
67
68
86
72
75
75
66
84
70
79
60
86
71
67
81
76
83
76
55
```

R Session:

```
R version 2.10.1 (2009-12-14)
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ISBN 3-900051-07-0

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  Natural language support but running in an English locale

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Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

[Workspace restored from /Users/andrejstreibergs/.RData]

> # Description of automated probability plot may be found in
> help(qqnorm)

> # SOME DATA IS CANNED IN  R.
> # precip  HAS ANNUAL RAINFALL OF 70 US CITIES (IN INCHES)
> precip
             Mobile              Juneau             Phoenix
               67.0                54.7                 7.0
        Little Rock         Los Angeles          Sacramento
               48.5                14.0                17.2
      San Francisco              Denver            Hartford
               20.7                13.0                43.4
         Wilmington          Washington        Jacksonville
               40.2                38.9                54.5
              Miami             Atlanta            Honolulu
               59.8                48.3                22.9
              Boise             Chicago              Peoria
               11.5                34.4                35.1
       Indianapolis          Des Moines             Wichita
               38.7                30.8                30.6
         Louisville         New Orleans            Portland
               43.1                56.8                40.8
          Baltimore              Boston             Detroit
               41.8                42.5                31.0
   Sault Ste. Marie              Duluth Minneapolis/St Paul
               31.7                30.2                25.9
            Jackson         Kansas City            St Louis
```
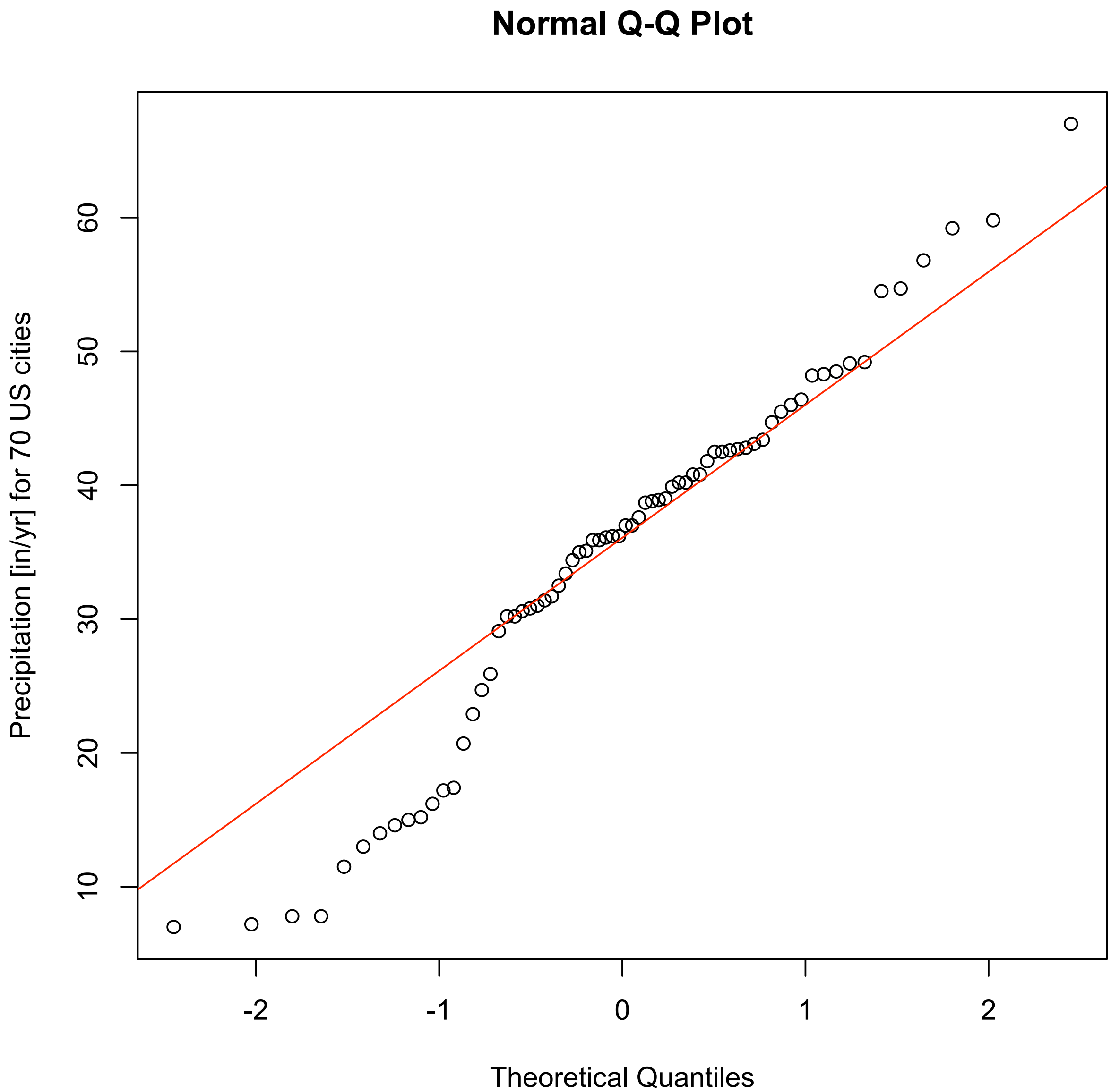
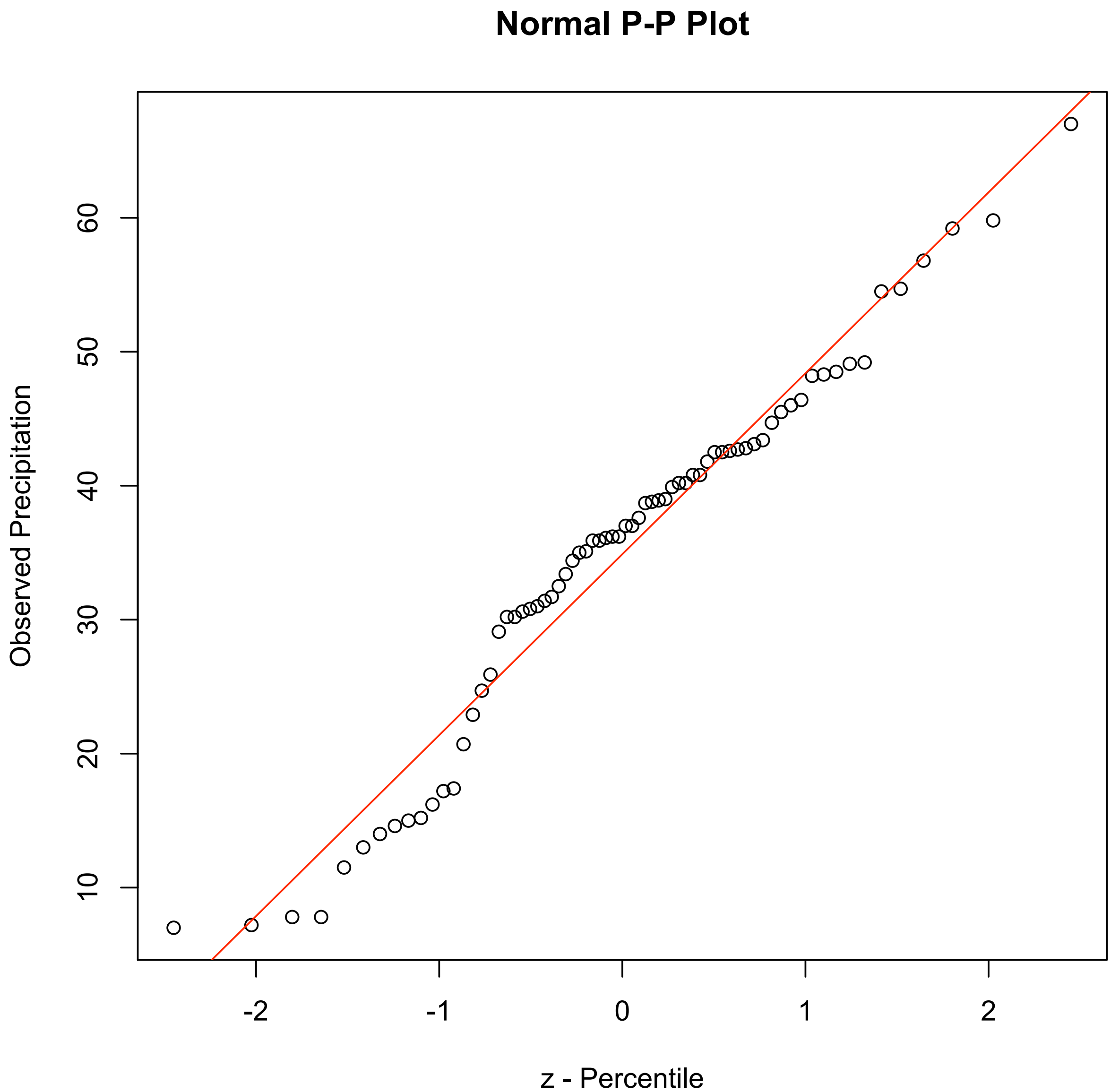

```
          49.2             37.0             35.9
   Great Falls            Omaha             Reno
          15.0             30.2              7.2
       Concord    Atlantic City      Albuquerque
          36.2             45.5              7.8
        Albany          Buffalo         New York
          33.4             36.1             40.2
     Charlotte          Raleigh          Bismark
          42.7             42.5             16.2
     Cincinati        Cleveland         Columbus
          39.0             35.0             37.0
 Oklahoma City         Portland     Philadelphia
          31.4             37.6             39.9
     Pittsburg       Providence         Columbia
          36.2             42.8             46.4
   Sioux Falls          Memphis        Nashville
          24.7             49.1             46.0
        Dallas          El Paso          Houston
          35.9              7.8             48.2
Salt Lake City       Burlington          Norfolk
          15.2             32.5             44.7
      Richmond   Seattle Tacoma          Spokane
          42.6             38.8             17.4
    Charleston        Milwaukee         Cheyenne
          40.8             29.1             14.6
      San Juan
          59.2

> # RUN THE AUTOMATED PROBABILITY PLOT qqnorm. ADD LINE VIA qqline
> # qqline  PUTS LINE THROUGH FIRST AND THIRD QUARTILES OF THE DATA.
> qqnorm(precip, ylab = "Precipitation [in/yr] for 70 US cities")
> qqline(precip,col=2)
```

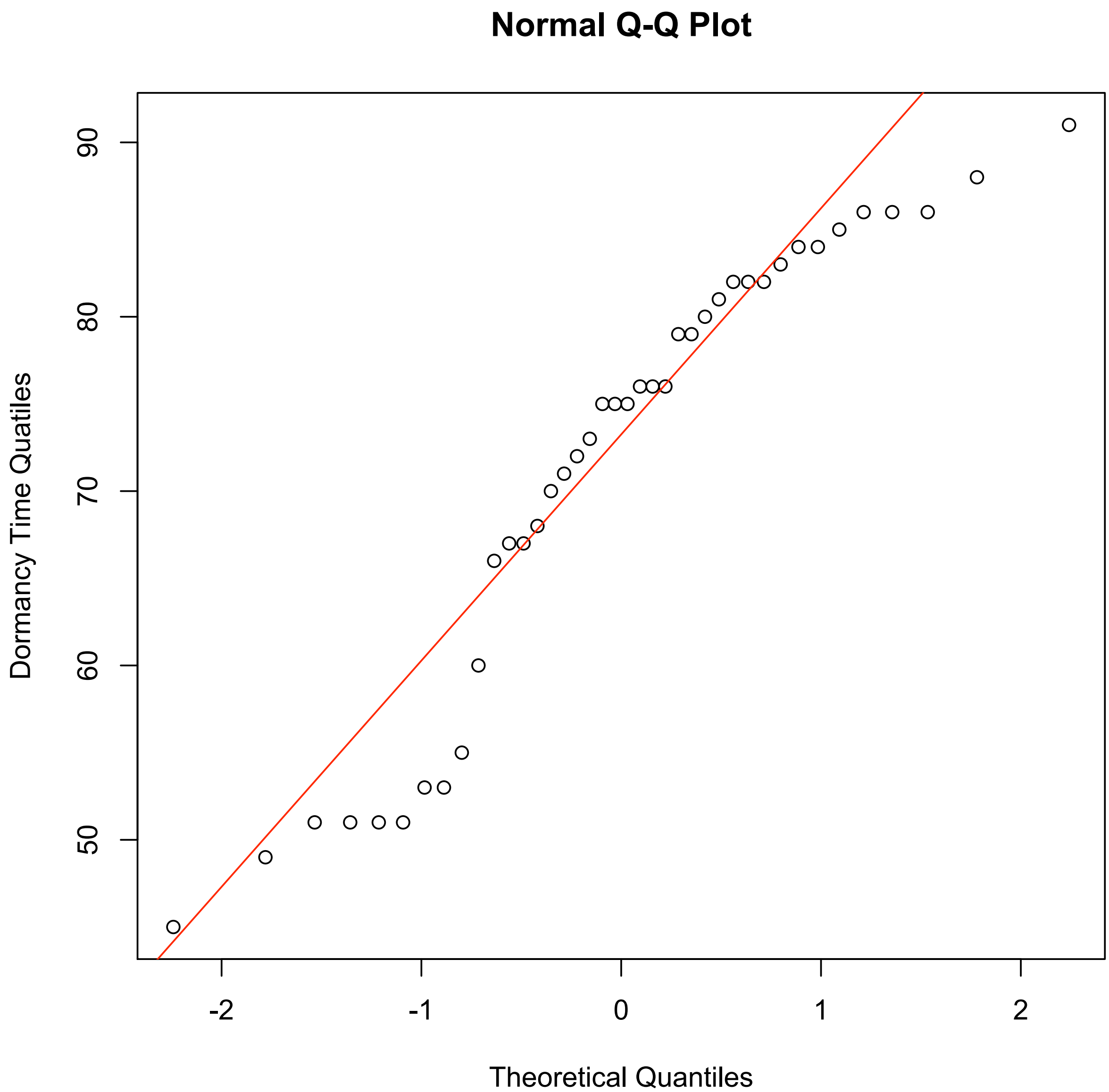

Normal Q-Q Plot
Precipitation [in/yr] for 70 US cities
10
20
30
40
50
60
-2
-1
0
1
2
Theoretical Quantiles

```
> ##########################################################################
> # DO THE PP PLOT BY HAND. GET SIZE OF SAMPLE
> n <- length(precip)

> # GET FRACTIONS (PERCENTILES) OF THE OBSERVED SAMPLE.
> Fi <- (1:70-0.5)/70
> Fi
 [1] 0.007142857 0.021428571 0.035714286 0.050000000 0.064285714 0.078571429
 [7] 0.092857143 0.107142857 0.121428571 0.135714286 0.150000000 0.164285714
[13] 0.178571429 0.192857143 0.207142857 0.221428571 0.235714286 0.250000000
[19] 0.264285714 0.278571429 0.292857143 0.307142857 0.321428571 0.335714286
[25] 0.350000000 0.364285714 0.378571429 0.392857143 0.407142857 0.421428571
[31] 0.435714286 0.450000000 0.464285714 0.478571429 0.492857143 0.507142857
[37] 0.521428571 0.535714286 0.550000000 0.564285714 0.578571429 0.592857143
[43] 0.607142857 0.621428571 0.635714286 0.650000000 0.664285714 0.678571429
[49] 0.692857143 0.707142857 0.721428571 0.735714286 0.750000000 0.764285714
[55] 0.778571429 0.792857143 0.807142857 0.821428571 0.835714286 0.850000000
[61] 0.864285714 0.878571429 0.892857143 0.907142857 0.921428571 0.935714286
[67] 0.950000000 0.964285714 0.978571429 0.992857143

> # GET THE CORRESPONDING  z - scores  BY SOLVING  Fi = Phi(Zi).
> Zi <- qnorm(Fi,0,1)
> Zi
 [1] -2.44999766 -2.02509955 -1.80274309 -1.64485363 -1.51975951 -1.41474643
 [7] -1.32336422 -1.24186679 -1.16787512 -1.09977866 -1.03643339 -0.97699540
[13] -0.92082298 -0.86741569 -0.81637496 -0.76737743 -0.72015662 -0.67448975
[19] -0.63018825 -0.58709060 -0.54505704 -0.50396537 -0.46370775 -0.42418819
[25] -0.38532047 -0.34702648 -0.30923489 -0.27188001 -0.23490082 -0.19824019
[31] -0.16184417 -0.12566135 -0.08964235 -0.05373932 -0.01790544  0.01790544
[37]  0.05373932  0.08964235  0.12566135  0.16184417  0.19824019  0.23490082
[43]  0.27188001  0.30923489  0.34702648  0.38532047  0.42418819  0.46370775
[49]  0.50396537  0.54505704  0.58709060  0.63018825  0.67448975  0.72015662
[55]  0.76737743  0.81637496  0.86741569  0.92082298  0.97699540  1.03643339
[61]  1.09977866  1.16787512  1.24186679  1.32336422  1.41474643  1.51975951
[67]  1.64485363  1.80274309  2.02509955  2.44999766

> # PUT THE OBSERVATIONS INTO INCREASING ORDER.
> # PLOT  SP  VS.  Zi.  DRAW THE BEST FIT LINE THIS TIME.
> SP <- sort(precip)
> plot(Zi,SP,ylab="Observed Precipitation",xlab="z - Percentile",main="Normal P-P Plot")
> abline(lm(SP~Zi),col=2)
```

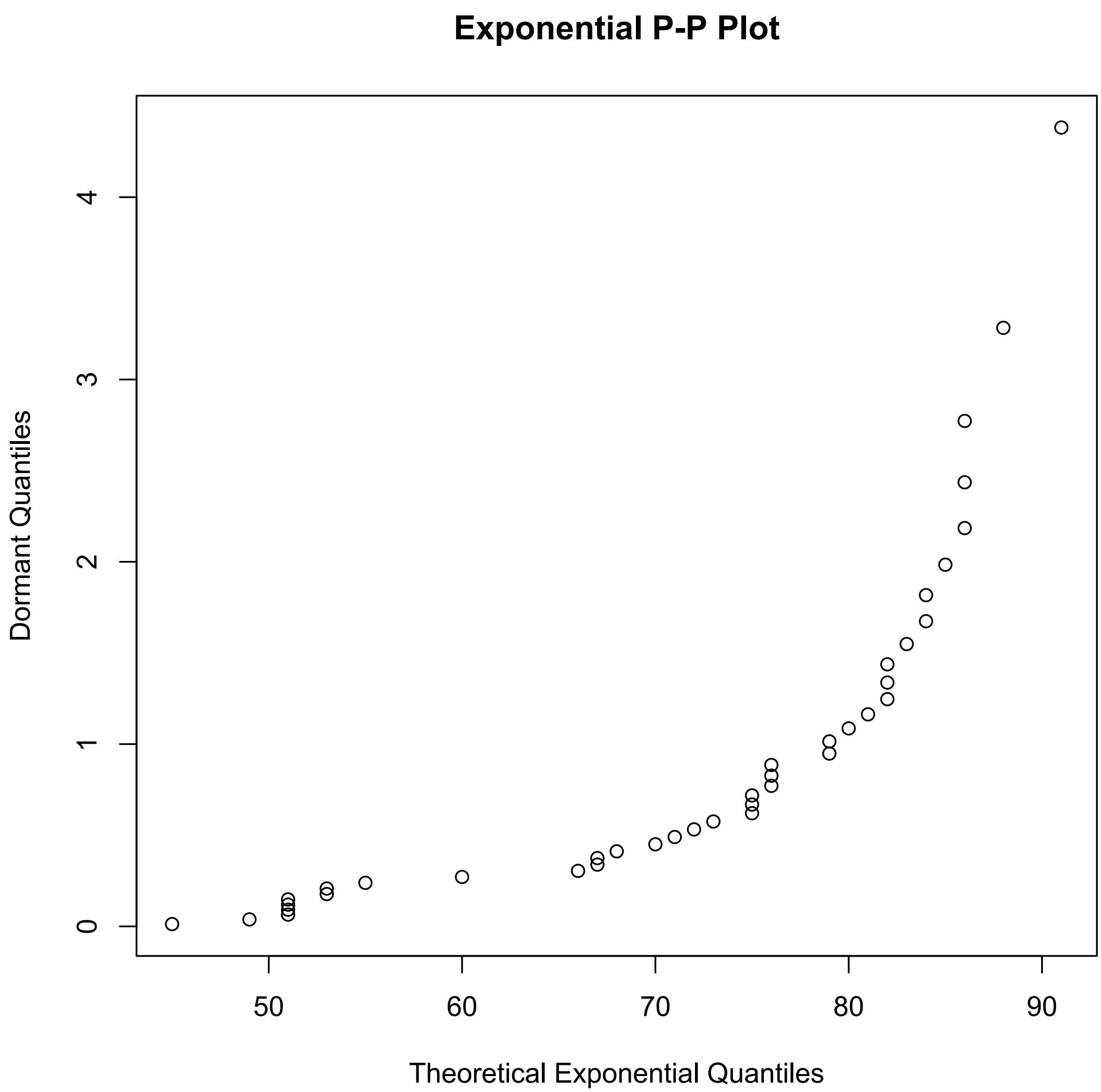

Normal P-P Plot
Observed Precipitation
10
20
30
40
50
60
-2
-1
0
1
2
z - Percentile

```
> ##########################################################################
> # READ DATA FROM FILE.  MAKE IT AVAILABLE TO  R.
> tt <- read.table("M3073GeyserData.txt",header=TRUE)
> attach(tt)
> tt
   Dormant
1       91
2       51
3       79
4       53
5       82
6       51
7       76
8       82
9       84
10      53
11      86
12      51
13      85
14      45
15      88
16      51
17      80
18      49
19      82
20      75
21      73
22      67
23      68
24      86
25      72
26      75
27      75
28      66
29      84
30      70
31      79
32      60
33      86
34      71
35      67
36      81
37      76
38      83
39      76
40      55
> Dormant
 [1] 91 51 79 53 82 51 76 82 84 53 86 51 85 45 88 51 80 49 82 75 73 67 68 86
[25] 72 75 75 66 84 70 79 60 86 71 67 81 76 83 76 55
```

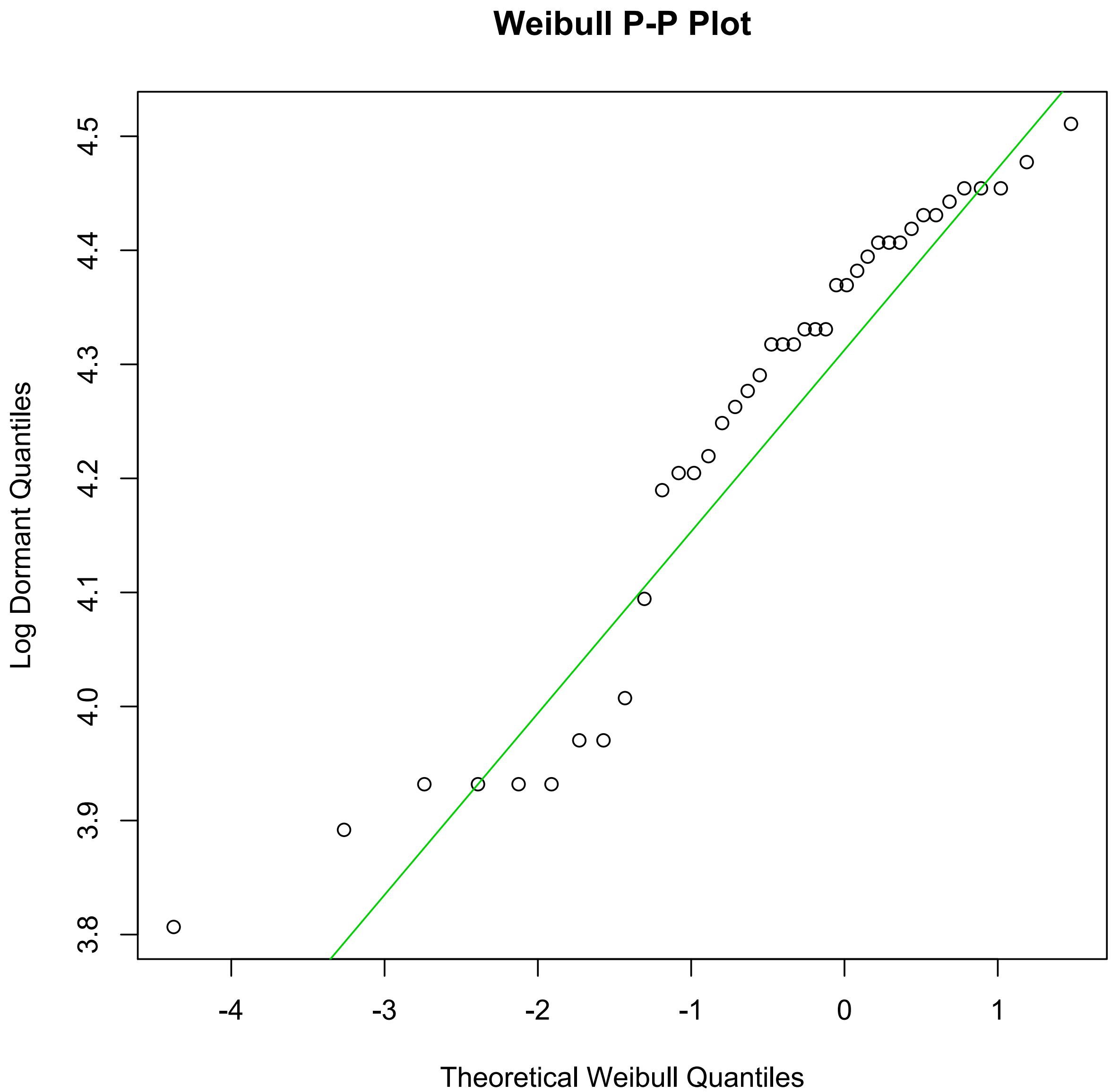
```
> USE CANNED PROB. PLOT AND LINE.
> qqnorm(Dormant,ylab="Dormancy Time Quantiles")
> qqline(Dormant,col=2)
```



```
> ##########################################################################
> NOT VERY NORMAL. TRY TO SEE IF IT IS EXPONENTIAL.
> n <- length(Dormant);n
[1] 40
> Fi<- (1:40-0.5)/40;Fi
 [1] 0.0125 0.0375 0.0625 0.0875 0.1125 0.1375 0.1625 0.1875 0.2125 0.2375
[11] 0.2625 0.2875 0.3125 0.3375 0.3625 0.3875 0.4125 0.4375 0.4625 0.4875
[21] 0.5125 0.5375 0.5625 0.5875 0.6125 0.6375 0.6625 0.6875 0.7125 0.7375
[31] 0.7625 0.7875 0.8125 0.8375 0.8625 0.8875 0.9125 0.9375 0.9625 0.9875

> # SORT  Dormant  INTO INCREASING ORDER
> SD <- sort(Dormant)
> SD
 [1] 45 49 51 51 51 51 53 53 55 60 66 67 67 68 70 71 72 73 75 75 75 76 76 76
[25] 79 79 80 81 82 82 82 83 84 84 85 86 86 86 88 91


> # lambda Qi  IS THE QUANTILE.  SOLVE EQUATION  Fi = 1 - exp(-lambda Qi)

> Qi <- -log(1-Fi);Qi
 [1] 0.01257878 0.03822121 0.06453852 0.09156719 0.11934676 0.14792013
 [7] 0.17733402 0.20763936 0.23889191 0.27115277 0.30448919 0.33897537
[13] 0.37469345 0.41173472 0.45020100 0.49020634 0.53187903 0.57536414
[19] 0.62082652 0.66845457 0.71846499 0.77110872 0.82667857 0.88551907
[25] 0.94803943 1.01473080 1.08618977 1.16315081 1.24653242 1.33750420
[31] 1.43758766 1.54881329 1.67397643 1.81707728 1.98413136 2.18480206
[37] 2.43611649 2.77258872 3.28341435 4.38202663

> plot(SD,Qi,ylab="Dormant Quantiles",xlab="Theoretical Exponential
  Quantiles",main="Exponential P-P Plot")
```


Exponential P-P Plot
Dormant Quantiles
0
1
2
3
4
50
60
70
80
90
Theoretical Exponential Quantiles

```
> ########################################################################
> # QUITE BOWED SO NOT EXPONENTIAL. TRY  WEIBULL  USING METHOD IN TEXT.

> # Qi/bets  IS THE QUANTILE.  SOLVE EQUATION  Fi = 1 - exp(-(-Qi/beta )^alpha)
> Eta <- log(-log(1-Fi))

> plot(Eta,log(SD),ylab="Log Dormant Quantiles",
  xlab="Theoretical Weibull Quantiles",main="Weibull P-P Plot")

> # ADD LINE. WORK OUT SLOPE  m  AND INTCPT  b  FROM  4th  TO  37th  POINT
> m<- (log(SD[37])-log(SD[4]))/(Eta[37]-Eta[4]);m
[1] 0.1592526
> b <- log(SD[4])-m*Eta[4];b
[1] 4.312548
> abline(b,m,col=3)
> # NOT SO WEIBULL EITHER, BUT SLIGHTLY "BETTER" THAN NORMAL OR EXPONENTIAL.
```


Weibull P-P Plot
Log Dormant Quantiles
3.8
3.9
4.0
4.1
4.2
4.3
4.4
4.5
-4
-3
-2
-1
0
1
Theoretical Weibull Quantiles