Homework for Math 3010 §1, Spring 2018

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Our text is by John Stillwell, Mathematics and its History, 3rd. ed., Springer, New York, 2010. Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on April 24, whichever comes first.

Your written work reflects your professionalism. Make answers complete, self contained and written in good English. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Monday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. The homework reader is ?? . Homework that is placed in his mailbox in JWB 228 before he picks it up not later than ?? pm Friday afternoon will be considered to be on time.

Please hand in problems A1 on Friday, January 12.

A1. Please hand in the following exercises from from Stillwell’s Mathematics and its History.

1.2.3 Show that any integer square leaves remainder 0 or 1 on division by 4.

1.2.4 Deduce from [1.2.3] that if $(a, b, c)$ is any Pythagorean triple then $a$ and $b$ cannot both be odd.

1.3.1 Deduce that if $(a, b, c)$ is any Pythagorean triple then

$$\frac{a}{c} = \frac{p^2 - q^2}{p^2 + q^2}, \quad \frac{b}{c} = \frac{2pq}{p^2 + q^2}.$$

for some integers $p, q$.

1.4.1 What has this figure to do with the Pythagorean Theorem?
1.4.2 In Book VI of *Elements*, Euclid gives the following argument for the Pythagorean Theorem based on similar triangles. Show that the three triangles in the figure are similar, and hence prove the Pythagorean theorem by equating ratios of corresponding sides.

![Diagram of triangles](image)

Please hand in problems B1–B3 on Friday, January 12.

B1. Please hand in the following exercises from Stillwell’s *Mathematics and its History*.

2.2.1 Show that for both cube and octohedron,

\[
\frac{\text{circumradius}}{\text{inradius}} = \sqrt{3}
\]

2.2.2 Check Pacioli’s construction. Consider the rectangle \([-\phi, \phi] \times [-1, 1]\). Suppose that three such rectangles are centered in the coordinate planes oriented so that the long directions align with the three axes as in the figure. Show that \(AB = AC = BC\). Recall that \(\phi = \frac{1}{2} + \frac{\sqrt{5}}{2}\) satisfies \(\phi^2 = 1 + \phi\).

![Diagram of rectangles](image)

B2. Show that the Golden section \(\phi\) is irrational.
B3. Check Euclid’s construction of the regular pentagon. Suppose that collinear points $A$, $B$, $C$ have distances $a = AB$, $\phi a = AC$. Construct the isosceles triangle $\triangle(BDC)$ with $a = BD = CD$. Let $M$ be the bisector between $B$ and $C$. Let the angle $\alpha = \angle(ACD)$.

a. Find the length of $DM$ using the triangle $\triangle(BMD)$

b. Show that $AD = AC$ using the triangle $\triangle(AMD)$.

c. Express the angles of triangle $\triangle(ABD)$ in terms of $\alpha$ and show that $\alpha = 72^\circ$. Thus it is the central angle of a sector of a regular pentagon. Hint: The sum of the interior angles of any triangle is $180^\circ$. 

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