Definitions and Formulas for Economics Applications
(from Applied Calculus, Hughes-Hallet et.al.)

Symbols

$q = \text{quantity}$

$p = \text{price}$

$P(q) = \text{profit earned when } q \text{ are sold}$

$R(q) = \text{revenue gained from selling } q$

$C(q) = \text{cost for producing and supplying } q$

$MR = R'(q) = \text{marginal revenue}$

$MC = C'(q) = \text{marginal cost}$

$AC = A(q) = \text{average cost}$

$E = \text{elasticity of demand}$

Profit is the difference between revenue and cost.

$P(q) = R(q) - C(q)$

Marginal Revenue is the revenue gained by selling one more unit.

$R'(100)$ is the additional revenue gained by producing the 100th unit.

Marginal Cost is the cost of producing one more unit.

$C'(100)$ is the additional cost paid when producing the 100th unit.

When the price can be defined as a function of quantity, or quantity as a function of price, revenue is equal to the product of quantity and price ($R = pq$).

The profit function has a critical point when the marginal revenue equals the marginal cost. Therefore, the maximum or minimum profit can occur when marginal revenue equals marginal cost. The maximum or minimum profit could also occur at endpoints, or points where the marginal cost or marginal revenue cannot be calculated. Critical points satisfy $R'(q) = C'(q)$.

If the cost of producing a quantity $q$ is $C(q)$, then the average cost, $AC = A(q)$, of producing a quantity $q$ is given by

$$AC = A(q) = \frac{C(q)}{q}$$
Important Distinction: The **average cost** is the cost per unit of producing a certain quantity. The **marginal cost** is the cost of producing the next unit.

Graphically, the **average cost** to produce \( q \) items is equal to the slope of the line drawn from the origin to the point \((q, C(q))\) on the cost curve.

\[
AC = A(q) = \frac{C(q) - 0}{q - 0}
\]

Relationship between **average cost** and **marginal cost**:

1. If \( MC < AC \), the \( AC \) is reduced by increasing production \((q)\).
2. If \( MC > AC \), the \( AC \) is increased by increasing production \((q)\).
3. If \( MC = AC \), the production level \((q)\) is a critical point of average cost.

The **elasticity of demand** for a product, denoted \( E \), is the magnitude of the ratio of fractional change in demand to fractional change in price. Again, \( q \) represents the quantity sold, and \( p \) represents the price of each unit. We have:

\[
E = \left| \frac{\Delta q/q}{\Delta p/p} \right| = \left| \frac{\Delta q}{q} \cdot \frac{p}{\Delta p} \right| = \left| \frac{p}{q} \cdot \frac{\Delta q}{\Delta p} \right| = \left| \frac{p}{q} \frac{dq}{dp} \right|
\]

when \( \Delta p \) is small. If \( E > 1 \), a one percent increase in price causes demand to drop by more than one percent, and a one percent decrease in price causes demand to increase by more than one percent. We say that demand is **elastic**. If \( 0 \leq E < 1 \), a one percent increase in price causes demand to drop by less than one percent, and a one percent decrease in price causes demand to increase by less than one percent. We say that demand is **inelastic**. In general, a large elasticity tells us that a small change in price will cause a large change in the number of sales.

Relationship between **elasticity** and **revenue**:

1. If \( E < 1 \), demand is inelastic and revenue is increased by raising price.
2. If \( E > 1 \), demand is elastic and revenue is increased by lowering price.
3. If \( E = 1 \) for a quantity \( q \), this is a critical point of the revenue function.