The Black & Brown Manufacturing Company is the only company that manufactures and sells stringless banjos - “the best banjos you’ll ever hear.” The demand for these banjos is modeled by the function

\[ D(p) = 1000 - \frac{p}{4} \]

where \( p \) is the price for each banjo. (Here, demand represents the number sold, or \( q \) from our usual notation.) The manufacturing cost of each banjo is $200 and, in addition, the Black & Brown Company pays a \( c\% \) commission to its salespeople (so if Jordan sells a banjo at price \( p \) dollars, he will receive a commission of \( \frac{c}{100}p \) dollars).

Let \( P = P(p) \) be the total profit, \( R(p) \) be the total revenue, \( C(p) \) be the total cost (manufacturing and commission), and \( S = S(p) \) be the total amount of sales commission income earned by the sales staff when the price per banjo is set at \( p \) dollars.

Case 1: commission rate \( (c) \) is 10%
Case 2: commission rate \( (c) \) is 15%

Answer parts (a)-(c) for your assigned case.
(a) Give formulas for \( P(p) \), \( R(p) \), \( C(p) \) and \( S(p) \).
(b) Determine the price \( p \) that will generate the most profit for the company. What is this maximum profit? What will be the corresponding total amount of commission income earned by the sales staff? What is the elasticity of demand at this price?
(c) Determine the price \( p \) that will generate the largest total amount of commission income for the sales staff. What is this maximum commission income? What will be the corresponding total profit earned by the company at this price? Will there be tension between the Company owners and the sales staff when it comes to setting the selling price of their banjos? What is the elasticity of demand at this price? Compare to the elasticity in part (b).

After hearing the results from both cases, answer part (d).
(d) Assume that the company will always set its price so as to maximize its profits (for the given level of commission). If the sales staff demands an increase in the commission rate from 10% to 15%, and the company grants their request, what will be the percentage change in the company’s profits? What will be the percentage change in the total amount of commission income for the sales staff? Would you recommend that the company agree to the commission rate increase?