Mathematics of Finance


Investing via Multiple Deposits

Description:

Three Scenarios:
(1) Future Value of an Ordinary Annuity
(a)

(b) Equation: \[ S = R \left( \frac{(1 + r_c)^N - 1}{r_c} \right) \]
(c) Variables:
   • \( S = \)
   • \( R = \)
   • \( N = \)
   • \( r_c = \)

(d) Types of Questions:

(2) Sinking Fund Payment Formula
(a)

(b) Equation: \[ R = S \left( \frac{r_c}{(1 + r_c)^N - 1} \right) \]
(c) Variables:
   • \( S = \)
   • \( R = \)
   • \( N = \)
   • \( r_c = \)

(d) Types of Questions:
(3) Future Value of an Annuity Due

(a) 

(b) Equation: \[ S_{due} = R(1 + r_c)\left(\frac{(1+r_c)^N - 1}{r_c}\right) = S(1 + r_c) \]

(c) Variables:
- \( S_{due} = \)
- \( R = \)
- \( r_c = \)
- \( N = \)
- \( S = \)

(d) Types of Questions:
Example 1: Suppose Jessica wants to create a college fund for her daughter, Olivia, who was just born. Jessica will deposit each of her annual $2,000 bonus checks in this college account on Olivia's birthday each year (i.e. at the end of each year of her life) for the next 18 years. The account earns 8% interest, compounded yearly. How much money will Olivia have toward college when she turns 18 years old?

Example 2: Suppose that you can afford to put $500 per month into an investment account to save toward your retirement. You regularly invest this amount for 20 years in an account that you predict will earn 7.5% interest compounded monthly, without taking to much risk. What will the account be worth upon your retirement?

Example 3: Mark has a debt of $28,000 to pay in 5 years. How much must he invest at the end of each quarter in an account earning 12% interest compounded quarterly to be able to pay off the debt?
**Example 4:** Find the future value of an account, earning 6% compounded monthly, with $150 deposited at the beginning of every month for 8 years.

**Example 5:** George and Mary have a friendly contest between them to see who can earn the most money in the next 5 years. They each can only invest a total of $10,000 in an account earning 10% interest compounded monthly, but neither of them has that much money up front. George decides to invest $5,000 at the beginning of the 5 years, and then add $85 at the end of each month to his account. Mary invests $170.00 at the end of every month for 5 years. Who has more money at the end of 5 years?
Example 6: You graduate from college at the age of 25 and because you're tired of living with little money, you decide not to invest toward your retirement for the first five years of your career. This allows you to enjoy more of your money for that time interval. After that five-year period, you invest $400 at the end of every month in an account earning 9% interest compounded monthly for the next 35 years.

(a) How much will your retirement account be worth at age 65?

(b) How much more would your account have been worth if you had only invested the $400 per month for the first eight years and then let the money sit there earning compound interest for the entire 40 years?

(c) How much would the account be worth if you had invested the $400 monthly for the entire 40 years?

Example 7: A $50,000 machine will have a market value of $6,000 in 15 years, which is the usable lifespan of this machine. A new machine is expected to sell for $68,000 at that time. The company sets up a sinking fund to pay for the new machine in the future. Payments are made to the fund twice each year. If the fund earns 7% interest compounded twice yearly, how much should each payment be?