REVIEW PROBLEMS FOR FINAL EXAM MATH 1210-002

Remarks: The final exam will have about 9 problems. It will take place Thursday, April 30, from 8AM-10AM in the usual classroom. You will be allowed 2 sides of an 8.5 by 11 inch sheet of notes, though calculators and other electronic devices will not be allowed. The exam will be comprehensive, and cover material through section 5.4 of the text that we covered in class, with a greater emphasis on material since the last exam. The following are some review problems. Solutions will be posted sometime on Tuesday. I also recommend looking over the midterm exams, as well as the review problems for those. I will be in my office on Tuesday 1-3PM and Wednesday, 10-12AM (though occasionally out for a few minutes).

1a. Find the limits \( \lim_{x \to 0} \frac{\sin(2x^2)}{x^2} \)

b. \( \lim_{x \to \infty} \frac{2x^3 + x + 1}{4x^3 + x^2 + 5} \)

2. Find the derivatives of the following functions:
   a. \( f(x) = \sin \sqrt{x^2 + 1} \)
   b. \( f(x) = \frac{\cos(2x)}{x^2 + 1} \)

3. For the function \( f(x) = x^4 - 2x^2 \), find the intervals on which \( f \) is increasing and decreasing, as well as concave up and concave down, and list the inflection points. Sketch a graph of \( f \).

4. Problem 14, page 174 of the text: A farmer wishes to fence off 3 identical adjoining rectangular pens, each with 300 square feet of area. What should be the length and width of each pen be, in order to use the least amount of fence material (that is, the total perimeter is as small as possible).

5. Assume that the following equation defines \( y \) as a function of \( x \). Use implicit differentiation to find \( y' \) in terms of \( x \) and \( y \). Find the equation of the tangent line at the point \((1,0)\)
   \[ y^2 + \cos(xy) + 3x^2 = 4 \]

6. Find the derivative of \( F \) where
   \[ F(x) = \int_{x^2}^{0} \cos(t^2) \, dt \]

7. Let \( R \) be the region in the plane bounded by the curves \( x = y^2 + 1 \) and \( x = y + 1 \). Sketch the region \( R \) and find its area.

8. Let \( S \) be the solid of revolution obtained by revolving the region \( R \) of problem 7, about the \( y \) axis. Find the volume of \( S \).

9. Let \( T \) be the solid of revolution obtained by revolving the region \( R \) of problem 7, about the \( x \) axis. Find the volume of \( T \).

10. Let \( C \) be the parametrized curve given by the equations \( x = 1 - 2\sin(t) \) and \( y = 2\cos(t) - 1 \), \( 0 \leq t \leq 2\pi \). Write down the integral for the length of \( C \). Find the length.

11. Let \( R \) be the region in the plane bounded by \( y = x \) and \( y = x^2 \). Suppose that the base of a solid is \( R \), and that cross sections of the solid perpendicular to the \( x \) axis are squares. Find the volume of the solid.