1. Below are sets with binary operations. Determine if each set is (or is not) a group and prove your answer.
   (a) For a fixed $n$, the numbers $\{0, 1, 2, \ldots, n-1\}$. The binary operation is multiplication modulo $n$.
   (b) The set of real-valued $n \times n$ matrices with positive determinant. The binary operation is matrix multiplication.
   (c) The set of real-valued $2 \times 2$ matrices with integer determinant. The binary operation is matrix multiplication.
   (d) The set of all finite products of the following matrices:
       \[
       \begin{bmatrix}
       0 & 1 \\
       1 & 0
       \end{bmatrix},
       \begin{bmatrix}
       1 & 2 \\
       0 & 1
       \end{bmatrix},
       \begin{bmatrix}
       1 & -2 \\
       0 & 1
       \end{bmatrix}.
       \]
       The binary operation is matrix multiplication.

2. Below are a number of sets with a potential binary operation. Verify whether or not it is indeed a binary operation. If it is a binary operation, prove that it is or is not a group.
   (a) Fix a group $G$ and consider the set $H = \{g \in G | ga = ag \forall a \in G\}$. The (potential) binary operation on $H$ is the binary operation from the group $G$.
   (b) The numbers $\{1, 2, \ldots, 7\}$. The (potential) binary operation is multiplication modulo $n$.
   (c) Fix a $3 \times 3$-matrix $A$. Consider the set of vectors $v$ in $\mathbb{R}^3$ such that $Av$ is zero. The (potential) binary operation is vector addition.
   (d) The set of real-valued $2 \times 2$ matrices with integer determinant. The (potential) binary operation is matrix addition.

3. For the groups you identified in problems 1. and 2. above, show whether each is (or is not) Abelian.

4. Suppose that $G$ is a group with the property that $g^2 = e$ for all $g \in G$. Prove that $G$ is Abelian.

5. Prove that in a group $G$, $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

6. Show that a group $G$ is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.

7. Find an example of a group $G$ and elements $a, b \in G$ such that $a^{-1}ba \neq b$.

8. Let $G$ be a group and fix $h \in G$. Define a function $\phi : G \to G$ defined by the rule $\phi(g) = hgh^{-1}$. Prove that $G$ is bijective.

9. Find an example of a group $G$ with both an element of finite order and an element of infinite order.