1) The parametrized curve $\sigma(t) = (1, t^3, t)$ intersects the surface $x^2 + z = 2$ in a point $P$.  
   a) Find coordinates of the point $P$.  
   b) Write a parametric equation of the line tangent to $\sigma(t)$ at the point $P$.  
   c) Write an equation of the tangent plane to the surface at the point $P$.  
2) Find the eigenvalues and the corresponding eigenvectors of the matrix  
$$
\begin{pmatrix}
1 & 1 \\
-2 & 4
\end{pmatrix}.
$$
3)  
   a) Sketch the region of integration for the integral  
   $$
\int_0^1 \int_0^1 \sqrt{y} \cos(x^4) dxdy.
$$
   b) Compute the integral (Hint: reverse the order of integration).  
4) Find the minimum and the maximum value of the function $f(x, y) = 4xy$ along the ellipse $x^2 + 4y^2 = 4$.  
5) Find and analyze the critical points of the function $f(x, y) = 2x^4 - x^2 + 3y^2$.  
6) Use a linear change of variables to evaluate the integral  
   $$
\int_S xy
dS
$$
where $S$ is a parallelogram with vertices $(0, 0), (2, 1), (1, 2)$ and $(3, 3)$.  
7) Evaluate the line integral by finding a potential function.  
   $$
\int_{(1, 1)}^{(2, 2)} (3x^2 + 2xy^2) dx + (3y^2 + 2x^2 y) dy.
$$
8) Let $C$ be the closed curve obtained by going from $(0, 0)$ to $(1, -1)$ along $y = -x$, then going to $(1, 1)$ along $x = 1$, and then coming back to $(0, 0)$ along $y = x$. Calculate  
   $$
\int_C x dy
$$
in two ways:  
   a) Directly.  
   b) Using the Green’s theorem.
9) Consider the vector field \( \mathbf{V} = xi + yj \). Calculate the flux of \( \mathbf{V} \) across the boundary of the triangle with vertices \((0,0), (1,0)\) and \((0,1)\) in two ways:
   a) Directly.
   b) Using the Divergence theorem.

10) Consider the vector field \( \mathbf{V}(x, y) = -yi + xj \). Let \( \mathbf{V}_\phi \) be the vector field obtained by rotating \( \mathbf{V} \) counter-clockwise for the angle \( \phi \). Find the flux of \( \mathbf{V}_\phi \) across the circle \( x^2 + y^2 = 1 \) two ways:
    a) Directly.
    b) Using the divergence theorem.

    Hint: You might want to do this for \( \mathbf{V} \) first. In general, for the second part, you will need to calculate \( \mathbf{V}_\phi \) explicitly. That isn’t too difficult! You just need to rotate the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) for the angle \( \phi \).