1) Let $K \subseteq L$ be two number fields. Let $A \subseteq B$ the corresponding rings of integers. Note that $A = B \cap K$. Let $L = \mathbb{Q}(\zeta)$ be the cyclotomic field where $\zeta = e^{\frac{2\pi i}{9}}$. Its ring of integers is $B = \mathbb{Z}[\zeta]$. Let $K = L \cap \mathbb{R}$, and let $A$ be the ring of integers in $K$. Prove that $A = \mathbb{Z}[x]$ where

$$x = \zeta + \bar{\zeta}.$$ 

Derive the cubic equation for $x$, using $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$. Compute the discriminant of $A$. Prove that $2A$ is a prime ideal. Combine this and Corollary 1 on page 58 of the book to prove that $A$ is a PID.

2) Let $A$ be a Dedekind domain with 2 maximal ideals $P$ and $Q$. Prove that $A$ is a principal ideal domain. Hint: apply Chinese Reminder Theorem to $A/P^2Q$ to construct a generator of $P$. 
