Math 2280  
Quiz 5

1. Determine the period and frequency of the simple harmonic motion of a body of mass 0.75 kg on the end of a spring with constant 48 N/m. 

Newton’s second law says that the mass of the body times its acceleration is equal to the applied force, which is supplied by the spring. Thus 

\[ mx'' = -kx \Rightarrow x'' + \frac{k}{m}x = 0. \]

The characteristic equation for this ODE is 

\[ r^2 + \frac{k}{m} = 0 \Rightarrow r = \pm \sqrt{\frac{k}{m}}i, \]

So the general solution is 

\[ x(t) = c_1 \cos \left( \sqrt{\frac{k}{m}} t \right) + c_2 \sin \left( \sqrt{\frac{k}{m}} t \right). \]

The period of oscillation is therefore 

\[ T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{48}{0.75}}} = \frac{\pi}{4} = 0.785 \]

with frequency 

\[ \frac{1}{T} = \frac{4}{\pi} = 1.273. \]

2. Write down the correct form of the particular solutions \( y_p \) for the following ODEs according to Rule 2. Do not solve for the coefficients. 

\( (a) \ y^{(3)} + y'' = x + e^{-x} \)
\( (b) \ y'' + 9y = 2 \cos 3x + 3 \sin 3x \)

(a) The characteristic polynomial for this equation is 

\[ r^3 + r^2 = 0 \Rightarrow r^2(r + 1) = 0 \Rightarrow r = 0, 0, -1, \]

so the complimentary solution is 

\[ y_c(x) = c_1 + c_2x + c_3e^{-x}. \]

According to Rule 2, we should take 

\[ y_p(x) = x^2[Ax^3 + Bx^2 + Cx + D + Ee^{-x}], \]

so that none of the terms in \( y_p \) are duplicated in \( y_c \). Note that our polynomial terms should have degree three (before multiplying by \( x^2 \)) so that the second derivative yields a first degree polynomial.

(b) The characteristic polynomial for this equation is 

\[ r^2 + 9 = 0 \Rightarrow r^2 = -9 \Rightarrow r = \pm 3i, \]

so the complimentary solution is 

\[ y_c(x) = c_1 \cos 3x + c_2 \sin 3x. \]

According to Rule 2, we should take 

\[ y_p(x) = x[A \cos 3x + B \sin 3x] \]

so that none of the terms in \( y_p \) are duplicated in \( y_c \).