3.90 Suppose that $n$ independent trials, each of which results in either 0, 1, or 2, with respective probabilities $p_0$, $p_1$, and $p_2$, are performed. Find the probability that outcomes 1 and 2 both occur at least once.

Let $A = \{\text{at least one 1 occurs}\}$, $B = \{\text{at least one 2 occurs}\}$.

\[
P(A \cap B) = 1 - P(A^c \cup B^c)
\]

\[
= 1 - \left[ P(A^c) + P(B^c) - P(A^c \cap B^c) \right]
\]

\[
= 1 - \left[ (1-p_1)^n + (1-p_2)^n - p_0^n \right]
\]

\[
= 1 - (1-p_1)^n - (1-p_2)^n + p_0^n
\]