Math Circle 2001–2002

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The purpose of these notes is to document the activities of the Math Circle this year, comment on what worked and what didn't, and propose some suggestions for the coming year. Many of the suggestions are based on the comprehensive evaluations provided by the participants.

Summary. I knew that the Math Circle had achieved a modest level of success during its inaugural year, but I was genuinely surprised by uniformly positive comments from the year end survey. More than one remarked that the Math Circle was the best math experience they had ever had. One person viewed the Math Circle as the highlight of their academic year. Another said the Circle was a kind of Salt Lake Math Club where one could learn new things in a distinctly social environment. Yet another said the only drawback of the program was that it met “only” two hours each week.

Clearly we have tapped into something significant. These kids are hungry for math enrichment, and the Math Circle provides exactly that which they are seeking. The program has gathered substantial momentum, and after learning from the missteps inherit in essentially starting the program from scratch this first year, I think we can add to that momentum.

Format. The Circle began October 3, 2001, and met from 4–6pm each subsequent Wednesday of the Fall and Spring terms. In order to maintain some level of continuity, two consecutive weeks were devoted to the same topic and were typically led by the same person. A typical session consisted of about an hour of the leader lecturing at the board and an hour of problem solving. It worked best when the lecturing and problem solving were intermingled.

Initially we closely modeled the Math Circle on the Berkeley template where the main thrust of each session was developing techniques to solve problems of the kind found on various international contests such as the International Math Olympiad. This proved problematic for several reasons, but perhaps most importantly neither I nor anyone involved in the program really had any experience with contests like the IMO. I didn’t have a catalog of techniques to share with the kids, and the the problems were too difficult to expect any participant (even the strongest) to make any progress on their own.

Based on this experience, beginning in the second term we essentially abandoned the contest problem focus. This proved much more successful, and opened up many new avenues (like topology, for instance, which certainly would never appear on any contest.) We still emphasized problem solving, only this time the problems were of our own devising and not contest-type problems. This worked very well.

Topics. Topics, in chronological order, were as follows:

  Summing polynomial sequences, tiling problems (Peter Trapa, 1 week)
  Induction (Renzo Cavalieri, 1 week)
  Magic squares, modular arithmetic (Jim Carlson, 2 weeks)
  Inclusion-Exclusion principle (Fletcher Gross, 2 weeks)
  Catalan numbers, bijective proofs (Mladen Bestvina, 2 weeks)
The game of 16, length and sign of a permutation, Robinson-Schested-Knuth algorithm
(Peter Trapa, 3 weeks)
Inversion in the plane (Zvezda Stankova, 1 week guest lecture)
Classification of surfaces (Renzo, 2 weeks)
Euler characteristic of polyhedra (Thom Pietraho, 2 weeks)
Hyperbolic geometry (Jen Taback, 1 week)
Five weeks were devoted to contests.
Ken Golden gave a presentation on the mathematics of sea ice as part of the last meeting’s
awards ceremony.

**Homework.** Assigning any kind of homework was usually a disaster. The kids seemed to
have too much already. The most one could reasonably expect is for them to think carefully
about one problem from week to week. (Even then, only about a third of them actually
did.) To encourage some thinking in between meetings, I would suggest giving a weekly
contest problem to be turned in at the beginning of the next week. The person with the
most correct solutions at the end of the term could be awarded a book prize. The problem
should be reasonably accessible, and perhaps could be tied to the current topic in the Circle.
Each session could begin with someone explaining their solution at the board. For this to
work, it would be important to limit this to the first 5–7 minutes of the session. If it took
much longer, it would be too distracting.

**What makes a good session?** One very positive aspect of this year’s program was the
atmosphere we managed to achieve and, independent of any particular topic or session
leader, it seems that this positive atmosphere is essential to the success of the program.
To use a single word, I would characterize the atmosphere this year as friendly. Renzo
and I got to know virtually all of the kids personally, and the relationships we developed
were less of student-to-teacher than simply friend-to-friend. For me, incidentally, this was
the most personally rewarding part of the program. I enjoyed watching the kids mature
mathematically.

This kind of friendly atmosphere immediately led to a freer exchange of ideas. Most kids
weren’t bashful about trying out their ideas or going to board to explain their solution even
if they ultimately turned out to be wrong. When an explanation was correct, we often
applauded.

Even more than the atmosphere, the session leader has the largest hand in controlling the
effectiveness of a particular session. I found that lecturing for more than 35 minutes was
generally a bad idea. These kids have been in school all day, and it’s hard for them to sit
through another traditional lecture. The best formula had the leader introduce new ideas in
an initial 20–30 minute segment, and then suggesting problems for the group to try. After
a 15 minute problem solving session (facilitated by Renzo and others walking around the
room helping individuals), volunteers presented their solution. After a short discussion, the
process would begin again with new ideas from the leader.

If a session leader wasn’t familiar with working with high school kids, I found it important
to have them give a brief overview to me before the actual meeting in order to head-off any
potential problems. In some cases (for instance, if a graduate student is giving his first ever
mathematical presentation in the Circle), I think it would be appropriate to have the leader
give a full dry run in front of an actual audience.
One final comment: I found it important to have a couple of people in the audience (like Renzo, myself, and Thom) who can gauge the level of understanding of the students and interrupt to clarify the session leader's exposition as appropriate. This also helps keep the atmosphere informal.

**Crowd Control.** Although generally a non-issue, occasionally there were some disruptive kids. I found this particularly infuriating (perhaps I need to mellow), but I suppose it comes with the territory. The kids are, with few exception, pretty good, and if they got out of line, all it took to correct the problem was a brief one-on-one conversation. But it's important to address this kind of problem almost immediately.

**Contests.** Initially we tried distributing a contest at the beginning of the month, and then collecting solutions at the end of the month. This was a disaster. Hardly anyone submitted solutions and those that did devoted little time to them.

We changed course and started monthly in-class contests that lasted an hour. Thom and I would determine winners while Renzo presented the solutions. Then I would return and award book prizes for the best solutions. This worked well.

Every single survey (even those from the weaker kids) commented positively on the monthly contests, and they should definitely be a permanent feature of the program.

**Suggestions for next year.** My main suggestion would be to continue leading sessions in the template we polished toward the end of the year and described above in the "What makes a good session?" section above. Renzo will be a crucial person to implement some level of continuity from this year to the next. He will be an important resource.

As far as topics are concerned, I would suggest an early installment of mathematical induction. Since we did that this year, many participants will know the rudiments of induction, but the spectrum of difficulty in induction problems is broad enough to engage all levels. The concept is important enough that it should be repeated. I might also suggest a few more sessions devoted to counting (one reinforcing inclusion-exclusion, for instance). Applications include computing dice and card probabilities. Learning to count is, by my estimation, an extremely important skill and perfect for the Circle.

One idea, which we didn’t explore at all this year, is preparing the kids for the State Math Contest. Several of the kids indicated in their surveys that although everything we did in the Circle was interesting, it didn’t have applications to the Math Contest. If there is a critical mass of interest, it may be worth devoting several sessions to contest preparation.

Based on the year-end surveys, meeting once a week for 2 hours was preferable to twice-weekly meetings. Many of these kids are over-programmed already, and even squeezing 2 additional hours into their schedule can be problematic. The current Wednesday meeting time seems to be the best compromise.

The homework section above suggests a kind of weekly contest. Some surveys suggested contests between teams or head-to-head competitions in the format of Jeopardy or other game shows. The team idea seems intriguing to me.

The surveys are valuable. I suggest one a term (as we did this year).

It is always difficult to round up enough faculty members to lead the Circle. The list of qualified and willing people is rather short. In addition to those we tapped this year, I would recommend trying Rossi, Mikhailkin, Hartenstein, Eneascu, Bertram, and Craw.
One potential resource is the group of VIGRE graduate students assigned to the Circle. This year, we had one or two on hand at each session to mingle with the kids while they were working problems. (In this context, I would like to single out Larsen Louder and Eric Cook for their substantial contributions above and beyond the call of their official duty.) Next year, I would suggest having the VIGRE students actually lead some of the sessions. This is a little delicate, since they would almost certainly need a lot of help not only in finding an appropriate topic, but also in polishing their presentation skills. Some kind of dry run of their sessions would likely need to be conducted. But I think having them lead sessions offers the VIGRE students an important opportunity to learn how to present mathematics to a general audience. It certainly fits nicely with the overall VIGRE mission.