Max/Min Word Problem Techniques

1. Draw a picture.
2. Label key elements in the diagram.
3. Label quantities that are not fixed with variables.
4. Find a function for the quantity that is to be minimized or maximized.
5. Eliminate variables using the conditions given so that the function found in Step 4 will be in terms of just one variable.
6. Find the domain of the function found in Step 5.
7. Maximize or minimize the function.
8. Make sure that you’ve answered the question that was asked and not just something related to the question.

A string $a$ feet long is to be cut into two pieces or left alone as one piece. One piece is used to make a circle while the other piece is used to make a square. How should the string be cut so that the sum of the areas is largest? How should the string be cut so that the sum of the areas is the smallest?

A poster is to consist of a picture (the picture has an area of 144 cm$^2$), right and left margins of 3 cm each, and top and bottom margins of 2 cm each. What should the dimensions of the picture be in order to minimize the total area of the poster?

A piece of property is to be fenced on the front and two sides. Available money is $400. Fencing for the sides costs $1 per foot and fencing for the front costs $1.60 per foot. What are the dimensions of the rectangular lot having largest area subject to these constraints?

A soft-drink manufacturer wants to fabricate aluminum cylindrical cans for its product. The can is to have a volume of 22 cubic inches (approximately 12 fluid ounces). Find the dimensions of the can that will require the least amount of aluminum.

An architect wants to design a window in the shape of a rectangle capped by a semicircle. If the perimeter of the window must be 24 feet, what dimensions should the architect choose for the window in order to admit the greatest amount of light?
A right circular cylinder is inscribed in a cone of height $H$ and radius $R$ so that the base of the cylinder lies on the base of the cone and the axis of symmetry of the cone and the cylinder are co-linear. Determine the dimensions of the cylinder with the largest possible volume. What is the maximum volume?

An isosceles triangle (i.e. two sides have the same length) has base 6 units and height 12 units. Find the maximum possible area of a rectangle that can be placed inside the triangle with one side on the base of the triangle. What are the dimensions of the rectangle(s) of maximum area?

![Diagram of a triangle with dimensions 6 and 12](image)

Find the points on the parabola $y = 6 - x^2$ closest to the point $(0,3)$.

Find the dimensions of the rectangle with base on the $x$-axis and right hand side on the line $x = 36$ and top-left corner on the graph of $y = \sqrt{x}$. Cite the appropriate test that justifies your answer.

![Diagram of a rectangle with graph of $y = \sqrt{x}$](image)