Local Linearization and its Application to Differentials

This is the general picture you should think of in regards to local linearization. We find the tangent line at a point and use the equation of the tangent line as an approximation for the function. As one application of local linearization we can consider $h$ as the change in $x$ (which is what $h$ really is) and let $dx = \Delta x$ which we can do because $dx$ is an independent variable and we can set it equal to whatever we want. This gives us that $\Delta y \approx dy = f'(x)dx$. You can see what everything looks like when we’ve made these substitutions by considering the next page.
\[
(x + \Delta x, f(x + \Delta x)) = (x + dx, f(x + dx))
\]

So we can see that if we let \(dx = \Delta x\) then we get that \(\Delta y \approx dy\) and we get an error term of

\[
\epsilon = f(x + dx) - f(x) - f'(x)dx = f(x + \Delta x) - f(x) - f'(x)\Delta x = \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} - f'(x)\right)\Delta x
\]

From this expression for \(\epsilon\) we can see that \(\epsilon \to 0\) as \(\Delta x \to 0\) therefore the error is approaching 0 which tells us that our approximation does get better and better as we shrink \(\Delta x\) to 0. Remember that this is just an application of the general process of local linearization \textit{not} local linearization itself.
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