Review problems for chapters 13 and 14:

The 3 problems are independent.

(1) Consider the solid region $R$ in 3-space above the surface $S$ with equation $z = x^2 + y^2$ and below the plane $P$ with equation $z = 1$. Evaluate the volume of $R$, by (a) using a double integral, and (b) using a triple integral. Find the area of the portion of the surface $S$ below $P$.

(2) Evaluate the following integral:

\[ \int_0^{\pi/2} \int_0^{\sqrt{2}} \int_0^{2yz} \sin \left( \frac{x}{y} \right) \, dx \, dy \, dz. \]

(3) Consider the vector field $\mathbf{F}(x, y, z) = < \cos x + 2yz, \sin y + 2xz, z + 2xy >$. (a) Is $\mathbf{F}$ conservative? (b) If yes, find a potential function $f$ for $\mathbf{F}$ (recall that this means that $\nabla f = \mathbf{F}$). (c) Evaluate the integral $\int_C \mathbf{F}(\mathbf{r}) \, d\mathbf{r}$, where $C$ is the path parametrized by $\mathbf{r}(t) = < R \cos t, R \sin t, t >$ (for a fixed $R > 0$) as $t$ varies from 0 to $4\pi$, and $\mathbf{r} = < x, y, z >$ is the position vector in 3-space.