The cosine and sine functions are defined to be the $x, y$ values of the intercept of a ray emanating from the origin at an angle $\theta$ from the $x$-axis with the unit circle.

$$t(x,y) = (\cos(t), \sin(t))$$

- $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cot \theta = \frac{1}{\tan \theta}$, $\sec \theta = \frac{1}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$

- With regards to a right triangle,

$$\sin t = \frac{\text{opp}}{\text{hyp}} \quad \cos t = \frac{\text{adj}}{\text{hyp}} \quad \tan t = \frac{\text{opp}}{\text{adj}}$$  \hspace{1cm} \text{(Recall sohcahtoa rhyme).}$$

- Sine and cosine are $2\pi$ periodic, whereas tangent is $\pi$ periodic.

- Sine function is odd, i.e., $\sin(-x) = -\sin(x)$.
  Cosine function is even, i.e., $\cos(-x) = \cos(x)$.
  Tangent function is odd.
- Graphing: simplify to the form \( f(\theta) = \pm A \sin[\omega(\theta - \delta)] + v \). \( A \) is the amplitude. \( \omega \) is the wave number and is related to the period of \( f(\theta) \) by \( p = \frac{2\pi}{\omega} \). The phase shift, \( \delta \), corresponds to the horizontal shift in the positive \( x \)-direction, that is, a shift \( \delta \) units to the right. A vertical shift can be taken into account by the \( v \) term.

- Inverse Trigonometric functions:
  \[
  x = \arcsin(y) \text{ if and only if } \sin(x) = y \text{ and } -\pi/2 \leq x \leq \pi/2
  \]
  \[
  x = \arccos(y) \text{ if and only if } \cos(x) = y \text{ and } 0 \leq x \leq \pi
  \]
  \[
  x = \arctan(y) \text{ if and only if } \tan(x) = y \text{ and } -\pi/2 \leq x \leq \pi/2
  \]

- Key trig identities:
  \[
  \sin^2(x) + \cos^2(x) = 1
  \]
  \[
  \sin(u - v) = \sin(u) \cos(v) - \sin(v) \cos(u)
  \]
  \[
  \cos(u - v) = \cos(u) \cos(v) + \sin(u) \sin(v)
  \]
  \[
  \sin^2(u) = \frac{1 - \cos(2u)}{2}
  \]
  \[
  \cos^2(u) = \frac{1 + \cos(2u)}{2}
  \]

  Derivable from above:
  \[
  \sin(2u) = 2 \sin(u) \cos(u)
  \]
  \[
  \cos(2u) = \cos^2(u) - \sin^2(u) = 2 \cos^2(u) - 1 = 1 - 2 \sin^2(u)
  \]

- Important triangles: