Math1220 Midterm 1 Review Problems
Answer Key

1. \( y = -4x + 4 + \frac{\pi}{2} \)

2. \( f^{-1}(x) = \frac{1 + 5\sqrt{x}}{2 - 2\sqrt{x}} \)

3. (a) \( y' = \frac{2\cos(3x)(-\sin(3x))(3)}{\cos^3(3x)} + \frac{3}{\sqrt{1 - (3x - 2)^2}} \)

(b) \( y' = (5x + 3)^{2x'}(4x \ln(5x + 3) + \frac{5(2x^2)}{5x + 3}) \)

(c) \( y' = \pi(1 + x^4)^{x'}(4x^3) + \pi^{1+x'}(\ln \pi)(4x^3) \)

(d) \( y' = -\text{sech}(2x)\tan(\cos(2x))(\sin(2x))(2) \)

(e) \( y' = \frac{3}{3x - 2} - 12x^{-7} + 12x^2 - 5\cos(5x) \)

(f) \( y' = e^{\frac{1}{3x}}\left(-\frac{1}{3x^2}\right) + \frac{1}{e^{\frac{1}{3x}}}(-3) \)

(g) \( y' = (x^3 - 1)^{\ln x}\left(\frac{1}{x}\ln(x^3 - 1) + \frac{3x^2(\ln x)}{x^3 - 1}\right) \)

(h) \( y' = \frac{-\sin x}{\sqrt{\cos x + 3^2 - 1}} \)

4. \( t = \frac{-10 \ln(0.08)}{\ln 2} \) years

5. Evaluate each integral.
   (a) \( x \arcsin(2x) + \frac{1}{2} \sqrt{1 - 4x^2} + C \)
   (b) \( 5 \ln|2x^2 + x - 7| + C \)
   (c) \( -5 \arctan(\ln x) + C \)
   (d) \( \frac{4^{-1} - 4^{-5}}{\ln 16} = \frac{255}{1024 \ln 16} \)
   (e) \( \frac{1}{2} y^2 \arctan(y) - y + \arctan(y) + C \)
   (f) \( -\frac{1}{\ln 2} \left(2^{\sqrt{1}/2} - 2\right) \)
   (g) \( -\frac{1}{3} \ln 4 \)
   (h) \( \frac{6^4 - 1}{\ln 36} \)
   (i) \( \frac{1}{2} \ln(e^{2x} + 5) + C \)
\[ \frac{5}{3} \arcsin(x^3) + C \]

\[ \frac{1}{4} \arctan\left(\frac{x^2}{2}\right) + C \]

\[ \frac{1}{4} \ln(x^4 + 4) + C \]

\[ \frac{3}{\ln 4} \left( x(4^x) - \frac{4^x}{\ln 4} \right) \]

\[ -\frac{\pi}{2} \]

6. \[ \frac{1}{14} \]

7. \[ f'(x) = \frac{\sin x + 1}{\cos^2 x} \] and since \( \sin x \) is always between -1 and 1, then \( 1 + \sin x \) must be between 0 and 2 (inclusive) which is always nonnegative. The denominator is also always positive on the interval \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \). This means that the derivative is always nonnegative in the given domain of the function. This implies that the function is monotonically increasing.

8. \[ \frac{1}{4} \]

9. \[ f'(x) = -\left( \frac{2}{1 + 4x^2} + 15(x - 1)^2 \right) \] which is always positive inside the parentheses since all coefficients are positive and the powers on \( x \) are even. Thus, the derivative is always negative which means the inverse function exists.

\[ (f^{-1})'(11) = \frac{1}{f'(0)} = -\frac{1}{17} \]

10.

(a) \[ \lim_{x \to \infty} \left( 1 + \frac{3}{x} \right)^{\frac{5x}{x}} = e^{\frac{15}{17}} \]

(b) \[ \lim_{x \to \infty} \left( 1 \right)^{\frac{5x}{x}} = 1 \]