Solution to Homework 11 (due date Apr.23)

20.18  a. One count is only 7 while the guidelines for using the large-sample method call for all counts to be at least 10.
b. For Wahtonka: 8 out of 137; for Warrenton: 28 out of 143.
c. We have $\hat{p}_1 = \frac{8}{137} = 0.0584$ and $\hat{p}_2 = \frac{28}{143} = 0.1958$, so the standard error for $\hat{p}_1 - \hat{p}_2$ is

\[
SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{0.0584(1-0.0584)}{137} + \frac{0.1958(1-0.1958)}{143}} = 0.0388.
\]

Thus the plus four 95% confidence interval is

\[
\left( (\hat{p}_1 - \hat{p}_2) - z^*SE, (\hat{p}_1 - \hat{p}_2) + z^*SE \right) = \left( (0.0584 - 0.1958) - 1.96 \cdot 0.0388, (0.0584 - 0.1958) + 1.96 \cdot 0.0388 \right) = (-0.2134, -0.0614).
\]

20.20 Let $p_1$ denote the proportion for Wahtonka and $p_2$ denote the proportion for Warrenton. The sample proportions are $\hat{p}_1 = \frac{7}{135} = 0.0519$ and $\hat{p}_2 = \frac{27}{141} = 0.1915$. The pooled sample proportion is $\hat{p} = (7 + 27)/(135 + 141) = 0.1232$. Standard error is

\[
SE = \sqrt{\hat{p}(1-\hat{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} = 0.0396.
\]

Hypotheses: $H_0 : p_1 = p_2$ v.s. $H_a : p_1 < p_2$.
Value of test statistic: $z = (\hat{p}_1 - \hat{p}_2)/SE = -3.53$.
$P$-value: < 0.0002.
Conclusion: We have strong evidence that drug use is lower in schools with testing programs.


20.30 Let $p_1$ denote the proportion of deaths among African miners and $p_2$ denote the proportion among European miners. The sample proportions are $\hat{p}_1 = \frac{223}{33809} = 0.006596$ and $\hat{p}_2 = \frac{1}{1}$,
7/1541 = 0.004543. The pooled sample proportion is \( \hat{p} = (223 + 7)/(33809 + 1541) = 0.006506 \). Standard error is

\[
SE = \sqrt{\hat{p}(1 - \hat{p})(\frac{1}{33809} + \frac{1}{1541})} = 0.0002094.
\]

Hypotheses: \( H_0 : p_1 = p_2 \) v.s. \( H_a : p_1 > p_2 \).

Value of test statistic: \( z = (\hat{p}_1 - \hat{p}_2)/SE = -0.98 \).

\( P \)-value: 0.1635.

Conclusion: We do not have enough evidence to conclude that the death rates are different.

20.31 Back of book.