Third Midterm Exam  
(MATH1070 Spring 2012)

Instructions: This is a one hour exam. You can use a notesheet. Calculators are allowed, but other electronics are prohibited.

1. [40pts] Multiple Choice Problems

1) A 99\% confidence interval for the average weight \( \mu \) of a population is computed from a random sample and found to be 6 \( \pm \) 2, or 4 to 8. We may conclude that A

(A) if we took many, many additional random samples, and from each computed a 99\% confidence interval for \( \mu \), approximately 99\% of these intervals would contain \( \mu \).

(B) there is a 99\% probability that \( \mu \) is between 4 and 8.

(C) there is a 99\% probability that the true mean is 6, and there is a 99\% chance that the true margin of error is 2.

(D) all of the above.

2) You plan to construct a confidence interval for the mean \( \mu \) of a Normal population with (known) standard deviation \( \sigma \). Which of the following will reduce the size of the margin of error? D

(A) Use a lower level of confidence.

(B) Increase the sample size.

(C) Reduce \( \sigma \).

(D) All of the above.

3) Suppose we want a 90\% confidence interval for the average amount of time (in minutes) spent per week on homework by the students in a large introductory statistics course at a large university. The interval is to have a margin of error of 2 minutes, and the amount spent has a Normal distribution with a standard deviation \( \sigma = 30 \) minutes. The number of observations required is closest to A

(A) 609.

(B) 865.

(C) 30.

(D) 25.
4) I collect a random sample of size \( n \) from a population and from the data collected compute a 95% confidence interval for the mean of the population. Which of the following would produce a new confidence interval with larger width (larger margin of error) based on these same data? **B**

- (A) Nothing can guarantee absolutely that you will get a larger interval. One can only say the chance of obtaining a larger interval is 0.05.
- (B) Use a larger confidence level.
- (C) Use a smaller confidence level.
- (D) Use the same confidence level, but compute the interval \( n \) times.

5) Is the mean age at which American children can first read now under 4 years? If the population of all American children has mean age of \( \mu \) years until they begin to read and standard deviation \( \sigma \) years, one would test which of the following null and alternative hypotheses to research this? **C**

- (A) \( H_0: \mu = 4 \) v.s. \( H_a: \mu \neq 4 \).
- (B) \( H_0: \mu = 4 \) v.s. \( H_a: \mu > 4 \).
- (C) \( H_0: \mu = 4 \) v.s. \( H_a: \mu < 4 \).
- (D) None of the above.

6) Suppose in a test of hypotheses, data are deemed to be significant (meaning rejecting null hypothesis) at level \( \alpha = 0.08 \), but not significant at level \( \alpha = 0.04 \). Which of the following is true about the \( P \)-value, \( P \), associated with this test? **B**

- (A) \( P > 0.08 \)
- (B) \( 0.04 < P < 0.08 \)
- (C) \( P < 0.04 \)
- (D) We can’t tell. It depends on the particular researcher’s level of significance used.

7) In testing hypotheses, if the consequences of rejecting the null hypothesis are very serious, we should **A**

- (A) conduct the test using a very small level of significance.
- (B) insist that the \( P \)-value be smaller than the level of significance.
- (C) conduct the test using a very large level of significance.
- (D) insist that the level of significance be smaller than the \( P \)-value.
8) In a test of hypotheses, the \( P \)-value is B

(A) the probability the null hypothesis is true.

(B) the probability, assuming the null hypothesis is true, that the test statistic will take a value at least as extreme as that actually observed.

(C) the probability, assuming the null hypothesis is false, that the test statistic will take a value at least as extreme as that actually observed.

(D) the probability the null hypothesis is false.
2. [10pts] A researcher used a new drug to treat 100 subjects with high cholesterol. For the patients in the study, after two months of treatment the average decrease in cholesterol level was 80 milligrams per deciliter (mg/dl). Assume that the decrease in cholesterol after two months of taking the drug follows a NORMAL distribution, with unknown mean $\mu$ and known standard deviation $\sigma = 20$ mg/dl. The researcher will construct a 90% confidence interval to estimate $\mu$. What is the researcher’s 90% confidence interval for $\mu$?

Since the population standard deviation is known, we will use $z$-procedure to construct a 90% confidence interval. A 90% confidence interval for the population mean $\mu$ is of the form

$$\left( \bar{x} - z^* \frac{\sigma}{\sqrt{n}}, \bar{x} + z^* \frac{\sigma}{\sqrt{n}} \right),$$

where $\bar{x}$ is the sample mean, $\sigma$ is the population standard deviation, $n$ is the sample size, and $z^*$ is the $z$-critical value corresponding to the confidence level 90%. Here $\bar{x} = 80$, $\sigma = 20$, and $n = 100$. From the standard normal table (or $t$-critical value table), we find that $z^* = 1.645$. Therefore the 90% confidence interval for the mean $\mu$ is

$$(76.71, 83.29).$$
3. [20pts] A snack food producer produces bags of peanuts labeled as containing 3 ounces. The actual weight of peanuts packaged in individual bags is NORMALLY distributed with mean $\mu$ and standard deviation $\sigma = 0.2$ ounces. As part of quality control, $n$ bags are selected randomly and their contents are weighed. The hypotheses of interest are $H_0 : \mu = 3$ v.s. $H_a : \mu > 3$. The inspector samples $n = 16$ bags and observes a sample mean weight of $\bar{x} = 3.1$ ounces. Based on these data, should the representative reject the null hypothesis at significance level $\alpha = 0.05$?

Since the population standard deviation is known, we will use $z$-test here. Therefore the test statistic is

$$z = \frac{\bar{x} - 3}{\sigma / \sqrt{n}} = \frac{3.1 - 3}{0.2 / \sqrt{16}} = 2.$$

The alternative hypothesis indicates that the $P$-value equals the right-tail probability corresponding to $z = 2$ for the standard normal distribution. From the table we see that the $P$-value equals 0.0228. Since the significance level is 0.05, we will reject the null hypothesis.
4. [10pts] The records of the 81 postal employees at a postal station in a large city showed that the average time these employees had worked for the postal service was $\bar{x} = 6$ years. The sample standard deviation for these 81 employees was $s = 5$ years. Assume that the times all U.S. Postal Service employees have spent with the postal service is NORMAL. Construct a 95% confidence interval for the mean time $\mu$ of all U.S. Postal Service employees have spent with the postal service.

Since the population standard deviation is unknown, we will use $t$-procedure to construct a 95% confidence interval. A 95% confidence interval for the population mean $\mu$ is of the form

$$(\bar{x} - t^* \frac{s}{\sqrt{n}}, \bar{x} + t^* \frac{s}{\sqrt{n}}),$$

where $\bar{x}$ is the sample mean, $s$ is the sample standard deviation, $n$ is the sample size, and $t^*$ is the $t$-critical value corresponding to the confidence level 95%. Here $\bar{x} = 6$, $s = 5$, and $n = 81$. Since the sample size is 81, the degrees of freedom of the $t$-distribution is 80. From the $t$-critical value table, we find that $t^* = 1.990$. Therefore the 95% confidence interval for the mean time $\mu$ of all U.S. Postal Service employees have spent with the postal service is

$$(4.894, 7.106).$$
5. [20pts] Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are NORMALLY distributed with mean \( \mu \). A representative of a consumer advocate group wishes to see if there is any evidence that the mean net weight is less than advertised and so intends to test the hypotheses \( H_0 : \mu = 14 \) v.s. \( H_a : \mu < 14 \). In order to do this, he selects 16 bags of this brand at random and determines the net weight of each. He finds the sample mean to be \( \bar{x} = 13.88 \) and the sample standard deviation to be \( s = 0.24 \). Based on these data, should the representative reject the null hypothesis at significance level \( \alpha = 0.01 \)? (Remind: the t-distribution is symmetric and the left-tail \( P \)-value corresponding to \( -|t| \) equals the right-tail \( P \)-value corresponding to \( |t| \).)

Since the population standard deviation is unknown, we will use \( t \)-test here. Therefore the test statistic is

\[
t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{13.88 - 14}{0.24/\sqrt{16}} = -2.
\]

Since the sample size is 16, the degrees of freedom for our \( t \)-statistic is 15. The alternative hypothesis indicates that the \( P \)-value equals the left-tail probability corresponding to \( t = -2 \) for the \( t \)-distribution with degrees of freedom 15. From the table we see that 1.753 < 2 < 2.131. Thus the \( P \)-value is between 0.025 and 0.05. Since the significance level is 0.01, we will not reject the null hypothesis.