6. (20 points) Solve the following equations:
(a) 
\[4x^2 + 6x = 4\]

Type of Equation: Quadratic

methods to solve: Factoring, QF, Completing the C

\[4x^2 + 6x - 4 = 0\]

QF:  \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[a = 4 \quad b = 6 \quad c = -4\]

\[x = \frac{-6 \pm \sqrt{36 - 4(4)(-4)}}{2(4)} = \frac{-6 \pm \sqrt{36 + 64}}{8} = \frac{-6 \pm \sqrt{100}}{8}\]

\[x = \frac{-6 \pm 10}{8} \Rightarrow x = -2, \quad x = \frac{1}{2}\]

(b) 
\[x(5x + 6) = 15\]

Expand the LHS of equation.
\[5x^2 + 6x = 15 \iff 5x^2 + 6x - 15 = 0\]

Solving using QF:  \[a = 5 \quad b = 6 \quad c = -15\]

\[x = \frac{-6 \pm \sqrt{36 + 4(5)(15)}}{10} = \frac{-6 \pm \sqrt{384}}{10}\]

\[x = \frac{-6 \pm 4\sqrt{91}}{10}\]
(c) Use log properties

\[
\log_2(3x - 2) - 2\log_2(x^2) = 0
\]

\[
= \log_2\left(\frac{3x - 2}{x^2}\right) = 0
\]

\[
\Rightarrow \frac{3x - 2}{x^2} = 2
\]

\[
\Rightarrow 3x - 2 = 2x^2
\]

\[
\Rightarrow 0 = x^2 - 3x + 2
\]

\[
\Rightarrow x = 1, x = 2
\]

(d) (Give an exact answer!)

\[
3e^{2x+1} - 1 = 11
\]

\[
3e^{2x+1} = 12
\]

\[
\ln e^{2x+1} = \ln(12)
\]

\[
2x+1 = \ln(12)
\]

\[
2x = \ln(12) - 1
\]

\[
x = \frac{\ln(12) - 1}{2}
\]
7. (15 points) Consider the function \( f(x) = -2(x - 5)^2 + 8 \) in parabola form. 
\( y = a(x-h)^2 + k \)

(a) What is the vertex of the parabola?
\[ (h, k) \text{ is vertex} \]
\[ \Rightarrow (5, 8) \]

(b) What is the axis of symmetry?
\[ x = h \text{ is axis of symmetry} \]
\[ \Rightarrow x = 5 \]

(c) What is the y-intercept?
occurs where \( x = 0 \).
\[ y = -2(-5)^2 + 8 = -2(25) + 8 = -50 + 8 = -42 \]
\[ y = -42 \]

(d) Is the parabola concave up or concave down? (How do you know?)
\[ \cap \text{ concave down} \]
\[ -2 = a < 0 \]

(e) Is the parabola stretched or shrunk vertically? (How do you know?)

Base Function: \( f(x) = x^2 \)

1. Shift to Right 5 units \( g(x) = (x-5)^2 \)
2. Stretch vertically 2 units \( h(x) = -2(x-5)^2 \) down
\[ a > 1 \]
3. Reflection vertically \( r(x) = -2(x+5)^2 \)
4. Shift + 8 units up \( l(x) = -2(x+5)^2 + 8 \)
(f) Graph the parabola.

$x$-intercepts: solve for roots.

\[ 0 = -2(x-5)^2 + 8 \]

\[ \Rightarrow -8 = -2(x-5)^2 \]
\[ \Rightarrow \sqrt{4 = (x-5)^2} \]
\[ \Rightarrow \pm 2 = x - 5 \]
\[ \Rightarrow x = 5 \pm 2 \quad \Rightarrow \quad x = 7, \quad x = 3 \]
8. (10 points) Consider the function

\[ f(x) = 3 \log(x - 4) \]

(a) Find the domain.

For what are acceptable inputs,
\[ x - 4 > 0 \implies x > 4 \]

\[ \forall x \in \mathbb{R}, x > 4 \]

(b) Find the \( x \)-intercept(s).

\( x \)-intercepts occur when \( f(x) = 0 \)

Set \( 0 = 3 \log(x - 4) \)

\[ 0 = \log \left( x - 4 \right) \implies 1 = x - 4 \implies x = 5 \quad (5, 0) \]

(c) Sketch the graph.
9. (10 points) Captain Bob wants to become a pirate. In order to realise his dream, he needs $40,000 to purchase and outfit a pirate ship. He has found an account that pays 7.2% interest compounded monthly. Bob intends to make payments into the account at the end of each month. How large does each payment need to be if he wants to have enough for a pirate ship in 5 years?

\[ r = 0.072 \]
\[ n = 12 \]
\[ t = 5 \text{ years} \]
\[ N = nt = 60 \text{ payments} \]
\[ S = 40,000 \]
\[ R = S \left( \frac{r_c}{(1+r_c)^N-1} \right) = 40,000 \left( \frac{0.006}{(1.006)^{60}-1} \right) \]

\[ R = 555.83 \]

10. (10 points) Captain Bill also wants to become a pirate, but he lacks Bob’s fiscal discipline. He outfits his pirate ship by taking out a $40,000 loan at 9% interest, compounded monthly. If Bill makes payments at the end of each month for five years, how much must each payment be?

\[ \text{loan } \Rightarrow \text{ group (4)} \]

\[ S = 40,000 \]
\[ r = 0.09 \]
\[ n = 12 \]
\[ t = 5 \]
\[ N = 60 \]
\[ r_c = \frac{0.09}{12} \]
\[ R = \frac{S}{1-(1+r_c)^{-N}} \]
\[ R = 40,000 \left( \frac{0.09}{12} \right) \left( \frac{1-(1+\frac{0.09}{12})^{-60}}{1-(1+\frac{0.09}{12})^{-60}} \right) \]

\[ R = 830.33 \]
11. (10 points) Paula received a trust fund inheritance of $60,000 on her 35th birthday. She plans to use it to supplement her income with 80 equal quarterly payments beginning on her 65th birthday. If money is worth 6.4% compounded quarterly, how much will she get at the end of each quarter?

\[
p = \frac{R(1-\left(\frac{1+r_c}{1+r}\right)^{-N})}{r_c(1+r_c)^m}
\]

\[t = 30 \text{ years}\]

\[P = \text{already have} \quad N = 80 \quad \Rightarrow \quad t_p = 20 \text{ years} \quad m = 120
\]

\[r = 0.064
\]

\[P = 60,000
\]

\[r_c = \frac{0.064}{4}
\]

\[R = \frac{P \cdot r_c \cdot (1+r_c)^m}{1-\left(\frac{1+r_c}{1+r}\right)^{-N}}
\]

\[= 60,000 \left(\frac{\frac{0.064}{4}\left(1 + \frac{0.064}{4}\right)^{120}}{1-\left(\frac{1 + \frac{0.064}{4}}{1}\right)^{-80}}\right)
\]

\[= 8,968.24
\]