NAME
Show all your work!

Problem 1. Write the following repeated decimal as a fraction in simplest form:

0.2439243924392439\ldots

Solution. Write

\begin{align*}
x &= \frac{2439}{10000} + \frac{2439}{100000000} + \frac{2439}{1000000000000} + \cdots \\
10000x &= 2439 + \frac{2439}{1000} + \frac{2439}{100000000} + \cdots \\
10000x &= 2439 + x \\
9999x &= 2439 \\
x &= \frac{2439}{9999} \\
x &= \frac{271}{1111}
\end{align*}

[8 points for 2439/9999, 10 points for 271/1111] □
**Problem 2.** Find all solutions to the following equation. Be sure to give an exact answer.

\[ x^2 + 2x = 5. \]

**Solution.** Write the equation as

\[ x^2 + 2x - 5 = 0, \quad (1) \]

and using the quadratic formula yields

\[
x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}
= \frac{-2 \pm \sqrt{24}}{2}
= 1 \pm \sqrt{6}
\]

[Partial credit: 1 point for (1), 1 point for trying to use the quadratic formula, 2 points for correct quadratic formula]

**Problem 3.** Find all \( x \) satisfying the inequality

\[ x^3 - 3x^2 + 2x \geq 0. \]

**Solution.** We first find the roots of \( x^3 - 3x^2 + 2x \):

\[
x^3 - 3x^2 + 2x = 0 \\
x(x^2 - 3x + 2) = 0 \\
x(x - 2)(x - 1) = 0.
\]

Thus the roots are \( x = 0, 1, 2 \).

Thus we have four regions to consider: \((-\infty, 0), (0, 1), (1, 2), (2, \infty)\). We will need to test the sign of the polynomial in each of these regions.

\[
(-1)^3 - 3(-1)^2 + 2(-1) = -6
\]
\[
\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) = \frac{3}{8}
\]
\[
\left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)^2 + 2\left(\frac{3}{2}\right) = -\frac{3}{8}
\]
\[
(3)^3 - 3(3)^2 + 2(3) = 6.
\]

Thus, the polynomial is non-negative in the intervals \([0, 1]\) and \([2, \infty)\).

[Partial credit: 2 points for trying to find roots, 2 points for testing in intervals between roots.]
Problem 4. Write \[ \frac{1}{x} - \frac{1}{x+1} \] as a rational expression (the quotient of two polynomials). What is the domain of this expression?

Solution.

\[
\frac{1}{x} - \frac{1}{x+1} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \\
= \frac{x+1 - x}{x(x+1)} \\
= \frac{1}{x(x+1)}.
\]

The domain is all \( x \) except \( x = 0, -1 \).

[partial credit: 7 points for writing as rational expression, 3 points for finding domain.]

Problem 5. Write the following polynomial in standard form, and find the degree and constant term:

\[ (x^2 - 2)(-2x + x^3 + 1) \]

Solution. We have

\[
(x^2 - 2)(-2x + x^3 + 1) = -2x^3 + x^5 + x^2 + 4x - 2x^3 - 2 \\
= x^5 - 4x^3 + x^2 - 2.
\]

Thus the degree is 5 and the constant term is \(-2\).

[partial credit: 6 points for standard form, 2 points for the degree, and 2 points for the constant term.]
Problem 6. Solve the following equation:

\[
\frac{x - 2}{x + 1} = \frac{x - 5}{x + 2}
\]

Solution.

\[
\frac{x - 2}{x + 1} = \frac{x - 5}{x + 2} \\
x - 2 = \frac{x - 5}{x + 2}(x + 1) \\
(x + 2)(x - 2) = (x - 5)(x + 1) \quad (2) \\
x^2 - 4 = x^2 - 4x - 5 \\
-4 = -4x - 5 \\
x = \frac{1}{4}
\]

[partial credit: 3 points for (2).]

Problem 7. You invest money at an (unusually good) annual interest rate that will triple your investment every 2 years. What is the interest rate?

Solution. We use the formula that the amount of money after \( t \) years with an initial investment of \( P \) is \( A = P(1 + r)^t \).

We solve for \( r \):

\[
3P = P(1 + r)^2 \\
3 = (1 + r)^2 \\
\sqrt{3} = 1 + r \\
r = \sqrt{3} - 1 \\
r \approx 0.732
\]

Thus \( r = 73.2\% \).

[partial credit: 4 points for setting up the equation (3).]
Problem 8. If
\[
\frac{x^a}{y^b} = \left( \frac{x^{-1/3}y^{2-1}}{yx^{2/3}} \right)^{1/2},
\]
then what are \( a \) and \( b \)?

Solution.
\[
\left( \frac{x^{-1/3}y^{2-1}}{yx^{2/3}} \right)^{1/2} = \left( x^{-1/3 - 2/3}y^{2-1} \right)^{1/2}
= \left( x^{-1}y \right)^{1/2}
= x^{-1/2}y^{1/2}
= \frac{x^{-1/2}}{y^{-1/2}}.
\]

Thus \( a = b = -1/2 \).

[partial credit: 5 points for either \( a \) or \( b \).]

Problem 9. Solve the following equation:
\[
|2x - 1| = x + 1.
\]

Solution. We have two equations. The first is:
\[
2x - 1 = x + 1
\]
\[
x = 2.
\]

The second is
\[
2x - 1 = -(x + 1)
\]
\[
3x = 0
\]
\[
x = 0.
\]

Thus the solutions are \( x = 0 \) and \( x = 2 \).

[partial credit: 4 points for only one correct solution.]
Distribution of Test Scores

The decimal point is 1 digit(s) to the right of the |

0   | 777
1   |
1   | 778
2   | 122
2   | 556667799
3   | 11133444
3   | 55777999
4   | 0000011113333334444
4   | 5556666888999
5   | 0000011234444
5   | 5666677777889999
6   | 00011111233334
6   | 5555555666666666667777788888888999
7   | 00111334444
7   | 555666689
8   | 001123
8   | 57

Min. 1st Qu. Median   Mean 3rd Qu.   Max.
7.00  41.00  56.00  53.45  67.00  87.00