SCALING,
SELF SIMILARITY
\[ \frac{4}{3} \]
FRACTALS

Figure 5.22: Blueprint of Barnsley's fern.

Figure 5.25: Barnsley's fern generated by an MRCM with only four lens systems.
Plate 2: Cast of a child's kidney, venous and arterial system, © Manfred Kage, Institut für wissenschaftliche Fotografie.

Plate 3: Broccoli Romanesco.
Plate 4: Wadi Hadramaut, Gemini IV image, © Dr. Vehrenberg KG.

Plate 5: Broccoli Romanesco, detail.
Plate 3.10.1
Three dimensional ferns.

Plate 11: "Carolina", fractal forgery, © K. Musgrave.
Plate 12: Fractal forgery of planet rise, © K. Musgrave.

Plate 13: "Ein kleines Nachtlicht", fractal forgery, stereoscopic image. View the left image with your right eye and the right image with your left eye. © K. Musgrave, C. Kolb, B.B. Mandelbrot.
PART I
CLASSICAL
SCALING

Bob transforms himself.
How does he do it??

"Bob" is a collection of points in the plane.

If we wish to translate Bob, say 2 units to the right and 1 unit up, we use the transformation function

\[ A([y]) = [x+2, y+1] = [x, y] + [2, 1] \]

\[ \uparrow \]

called the translation vector

\[ A(Bob) \text{ means the set of } A([y])'s, \text{ where } [y] \text{ is in Bob.} \]

Easy moves

- translation \[ A([y]) = [x] + [e] \]

- uniform scaling, then translation (say by \( R \)) \[ A([y]) = [Rx] + [e] \]

- nonuniform scaling, then translation

- turning Bob over?
Bob transforms himself.

WHAT FORMULAS?
What does scaling do to
- Areas, volumes, lengths:

Enclosed areas?
circumferences?

Enclosed volumes?
sphere surface areas?

radius 1
radius $R$
radii $a, b, c$
Classical Scaling:

- Why is the Pythagorean Theorem true? \( a^2 + b^2 = c^2 \)
  
  Hint: use area scaling, under uniform scaling:
Hardean transformations:
(more general)
\[ A([x, y]) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax + cy + e \\ bx + dy + f \end{bmatrix} \]
\[ \iff \text{transformations of this sort are called Affine} \]

includes the easy transformations:

translation:
\[ A([x, y]) = [x, y] + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} [x] + \begin{bmatrix} 2 \end{bmatrix} \]

scaling and translation:
\[ A([x, y]) = \begin{bmatrix} 2x \\ 5y \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} [x] + \begin{bmatrix} 2 \end{bmatrix} \]

reflection:
\[ A([x, y]) = [-x, y] = \begin{bmatrix} -1 \\ 0 \end{bmatrix} [x] + \begin{bmatrix} 0 \end{bmatrix} \]

etc.

also rotations

rotate by \( \Theta \) radians counterclockwise:
\[ A([x, y]) = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} [x, y] \]
Geometry of affine transformations

$$A([x]) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

**Theorem** Let $A$ be an affine map. Then

1. For any two points $P, Q$ and their midpoint $M$,
   The midpoint of $A(P), A(Q)$ is $A(M)$

(2) Affine maps transform lines into lines
(3) Affine maps transform rectangles into parallelograms. (a parallelograms)
They transform rectangular grids into parallelogram grids.
Affine transformation template

\[ A([x \ y]) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ax + cy + e \\ bx + dy + f \end{bmatrix} \]

This point distorts Bob

This point translates him

\[ A([x \ y]) = \begin{bmatrix} a + e \\ b + f \end{bmatrix} \]
Bob & L-box transform themselves

1. Find formulas for the two affine maps which one shows.

2. Show where the L-box is transformed to by

\[ A(x, y) = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -4 \\ -3 \end{bmatrix} \]
How do affine transformations affect area?

How are the area of "Bob" and "A(Bob)" related?

- Translations don't change area, so assume \([f] = [0]\)

\[
A([x]) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} [x]
\]