Math 2270-2
Second Exam Review Information
October 26, 2001

I have reserved JWB 208 tomorrow (Saturday), from 1-2:30 in the afternoon, for a problem session.

The exam will cover section 3.4, 4.1-4.3, 5.1-5.5. In addition to being able to do the computations from these sections, you should know key definitions, the statements of the main theorems, and why they are true. The exam will be a mixture of computational and theoretical questions. As on the first exam, there will (also) be some true-false questions, see e.g. those at the ends of chapters 3 and 5.

One way to organize the topics is as follows:

**Linear Spaces, 3.4-4.3** (also called vector spaces)

**Definitions:**
- Linear space
- subspace
- Linear transformation
- domain
- codomain
- kernel
- image
- rank
- nullity
- linear isomorphism
- linear combination
- span
- linear dependence, independence
- basis
- dimension
- coordinates with respect to a basis
- matrix of a linear transformation

**Theorems:**
- results about dimension: e.g. if \( \text{dim}(V)=n \), then more than \( n \) vectors are ?, fewer than \( n \) vectors cannot ?; \( n \) linearly independent vectors automatically ?, \( n \) spanning vectors automatically are ?
- also, if a collection of vectors is dependent, it may be culled without decreasing the span; if a vector is not in the span of a collection of independent vectors, it may be added to the collection without destroying independence.
- the kernel and image of linear transformations are subspaces.
- rank plus nullity equals ?
- A linear transformation is an isomorphism if and only if ?
- Isomorphisms preserve ?

**Computations:**
- Check if a set is a subspace
- Check if a transformation is linear
- Find kernel, image, rank, nullity of a linear transformation
- Check if a set is a basis; check spanning and independence questions.
Orthogonality (Chapter 5)

**Definitions:**
- orthogonal
- magnitude
- unit vector
- orthonormal collection
- orthogonal complement to a subspace
- orthogonal projection
- angle
- correlation coefficient (not on exam, but interesting)
- orthogonal transformation, matrix
- transpose
- least squares solutions to $Ax=b$
- inner product spaces

**Theorems**
- Pythagorean Theorem
- Cauchy-Schwarz Inequality
- Any basis can be replaced with an orthonormal basis (Gram Schmidt)
- Algebra of the transpose operation
- symmetric, antisymmetric
- algebra of orthogonal matrices
- QR factorization

**Computations**
- coordinates when you have an orthonormal basis
- Gram-Schmidt
- orthogonal projections
- least squares solutions
- application to best-line fit for data
- matrix for orthogonal projection