• Go over course information on syllabus and course homepage:

   http://www.math.utah.edu/~korevaar/2250fall16

• Notice that there is homework due this Friday, and that your first lab meeting is this Thursday.

Then, let's begin!

• What is an \( n \)th order differential equation (DE)?

   any equation involving a function \( y = y(x) \) and its derivatives, for which the highest derivative appearing in the equation is the \( n \)th one, \( y^{(n)}(x) \); i.e. any equation which can be written as

   \[
   F(x, y(x), y'(x), y''(x), \ldots, y^{(n)}(x)) = 0.
   \]

Exercise 1: Which of the following are differential equations? For each DE determine the order.

a) For \( y = y(x) \), \( (y'''(x))^2 + \sin(y(x)) = 0 \)

b) For \( x = x(t) \), \( x'(t) = 3 x(t)(10 - x(t)) \).

c) For \( x = x(t) \), \( x'^2 = 3 x (10 - x) \).

d) For \( z = z(r) \), \( z'''(r) + 4 z(r) \).

e) For \( y = y(x) \), \( y' = y^2 \).
A solution function $y(x)$ to the differential equation $F(x, y, y', y''', y^{(n)}) = 0$ defined on some interval $I$ is any function $y(x)$ which makes the differential equation a true equality for all $x$ in $I$.

A solution function $y(x)$ to a first order differential equation $F(x, y, y') = 0$ on the interval $I$ which also satisfies $y(x_0) = y_0$ for a specified $x_0 \in I$ and $y_0 \in \mathbb{R}$ is called a solution to the initial value problem (IVP)

$$F(x, y, y') = 0$$
$$y(x_0) = y_0$$

Exercise 2: Consider the differential equation $\frac{dy}{dx} = y^2$ from (1e).

2a) Show that functions $y(x) = \frac{1}{C - x}$ solve the DE (on any interval not containing the constant $C$).

2b) Find the appropriate value of $C$ to solve the initial value problem

$$y' = y^2$$
$$y(1) = 2.$$
2c) What is the largest interval on which your solution to (b) is defined as a differentiable function? Why?

2d) Do you expect that there are any other solutions to the IVP in 2b? Hint: The graph of the IVP solution function we found is superimposed onto a "slope field" below: The line segments at points (x, y) have values $y^2$, because solutions graphs to the differential equation $y' = y^2$ will have slopes given by the derivatives of the solutions $y(x)$. This might give you some intuition about whether you expect more than one solution to the IVP.
- **important course goals**: understand some of the key differential equations which arise in modeling real-world dynamical systems from science, mathematics, engineering; how to find the solutions to these differential equations if possible; how to understand properties of the solution functions (sometimes even without formulas for the solutions) in order to effectively model or to test models for dynamical systems.

In fact, you've encountered differential equations in previous mathematics and/or physics classes:

- 1<sup>st</sup> order differential equations: rate of change of function depends in some way on the function value, the variable value, and nothing else. For example, you've studied the population growth/decay differential equation for $P = P(t)$, and $k$ a constant, given by
  \[ P'(t) = kP(t) \]
and having applications in biology, physics, finance.

- 2<sup>nd</sup> order DE's: Newton's second law (change in momentum equals net forces) often leads to second order differential equations for particle position functions $x = x(t)$ in physics.

**Exercise 3**: The mathematical model in which the time rate of change of a population $P(t)$ is proportional to that population is expressed mathematically as
\[ \frac{dP}{dt} = kP \]
where $k$ is the proportionality constant.

3a) Find all solutions to this differential equation by using the chain rule backwards.
3b) The method of "separation of variables" is taught in most Calc I courses, and we'll cover it in detail in section 1.4. It's an algorithm which hides the "chain rule backwards" technique by treating the derivative $\frac{dP}{dt}$ as a quotient of differentials. Recall this magic algorithm to recover the solutions from (3a).
Exercise 4) Newton's law of cooling is a model for how objects are heated or cooled by the temperature of an ambient medium surrounding them. In this model, the body temperature $T = T(t)$ changes at a rate proportional to the difference between it and the ambient temperature $A(t)$. In the simplest models $A$ is constant.

a) Use this model to derive the differential equation

$$\frac{dT}{dt} = -k(T - A).$$

b) Would the model have been correct if we wrote $\frac{dT}{dt} = k(T - A)$ instead?

c) Use this model to partially solve a murder mystery: At 3:00 p.m. a deceased body is found. Its temperature is 70°F. An hour later the body temperature has decreased to 60°F. It's been a winter inversion in SLC, with constant ambient temperature 30°F. Assuming the Newton's law model, estimate the time of death.