A differential equation is an equation
\[ F(x, y, y', y'', \ldots, y^{(n)}) = 0 \]
where \( x \) is a name for the variable (domain an interval of \( \mathbb{R} \))
(sometimes use "\( t \)" for this variable)
and \( y(x) \) is a function, with derivatives \( y', y'', \ldots, y^{(n)} \)
the order of the DE is the highest order deriv which appears ("\( n \)"
• **goal**: Find the functions \( y(x) \) which make the DE a true equality.
  (This is called solving the DE).
  Perhaps \( y(x) \) will be required to satisfy further conditions as well,
  e.g. initial conditions.

• Where do DE’s come from?
  - They are often the result of mathematical models in science, engineering,
    everywhere!

Examples you have seen

\[ \frac{dP}{dt} = kP \]
\( k \) constant

- **Model**: “Rate of change of \( P(t) \) is proportional to \( P(t) \)”

\( k > 0 \) simple population growth
\( k < 0 \) radioactive (or other) decay

HW due 9/4 (circled means hand in)
1.1 3, 4, 7, 15, 16, 19, 20, 27, 30, 37, 38, 39
1.2 5, 7, 10, 14, 20, 25, 26, 30, 49
1.3 3, 6, 9, 11, 12, 14, 21, 29, 32, 33

Examples you have seen

\[ \frac{dP}{dt} = kP \]
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Chain rule backwards:
Differentials:
(2) Newton's Law of cooling (or heating)

\[ \frac{dT}{dt} = k(A - T) \]

- \( T(t) \) = temperature at time \( t \) of object
- \( A \) = constant ambient temperature

"how fast temp. changes is proportional to difference between \( T(t) \) and \( A \)"

(3) springs:

- \( m \) x'' = -k x - \mu x
  - Hooke's constant
  - coeff. of friction
  - if \( \mu = 0 \) get simple harmonic motion; try sin solns.

(4) projectiles:

\[ m \frac{d^2y}{dt^2} = -mg \]

- \( y(t) \) = height at time \( t \)
- \( g = \) accel. of gravity = 9.8 \( \text{m/sec}^2 \)

\[ y'' = -g \]

- \( y' \) =

```
Solve it!
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Example: (a) Show \( y(x) = \frac{1}{C-x} \) solves \( \frac{dy}{dx} = y^2 \)

(b) solve the initial value problem

\[
\begin{cases}
\frac{dy}{dx} = y^2 \\
y(1) = 2
\end{cases}
\]
Murder mystery

65° = A
3 a.m. body temp 85°
4 a.m. " " 80°

When did body die, estimate with Newton's Law of cooling.

\[
\frac{dT}{dt} = k (A-T)
\]

\[
\frac{dT}{A-T} = k \; dt
\]