These notes discuss material from sections 5.5 and 5.6 of the text, in the context of Maple. They are online at our Maple page, go to www.math.utah.edu/~korevaar/2250fall03/2250maple.html.

**Part 1: checking homework problems on Maple**

Here's a homework problem worked out in Maple, **#12 from section 5.5**. This problem was fairly painful to work by hand. For more information on the commands which are used, use the Help files.

```
> with(DEtools): #DE command library
> f:=x->2-sin(x): #inhomogeneous term
> deqtn:=diff(y(x),x,x,x)+diff(y(x),x)=f(x):
    #the differential equation, #12 page 337.
> dsolve(deqtn,y(x));

y(x)=\text{-}C2 \cos(x)+\text{-}C1 \sin(x)+\frac{1}{2} x \sin(x)+2 x +C3
```

Discarding the pieces which solve the homogeneous equation, the simplest particular solution we see is

```
> yp:=x->1/2*x*sin(x)+2*x;
```

The method of guessing as modified for problems when the guess for yp involves homogeneous equation solutions, would have had us to a guess of the form \( yp=Ax + x(B\sin(x)+C\cos(x)) \). The algebra in computing \( L(yp) \) was messy. We could check the algebra part of our work as well as the final answer:

```
> yp:=x->A*x+x*B*sin(x)+x*C*cos(x);
> diff(yp(x),x,x,x)+diff(yp(x),x)=2-sin(x);
```

Equating coefficients we see that \(-2B=-1\), \(C=0\), and \(A=2\). This leads to \( yp=(1/2)x\sin(x)+2x \), as before.

**How about #37?**

```
> f:=x->1+x*exp(x): #right hand side, then DE:
> deqtn:=diff(y(x),x,x,x) -2*diff(y(x),x,x)
    + diff(y(x),x) =f(x);
> ics:=y(0)=0, D(y)(0)=0, D(D(y))(0)=1:
    #initial conditions...see help files for syntax!
> ans37:=dsolve({deqtn,ics},y(x));

\[
\begin{align*}
\frac{d^3}{dx^3} y(x) - 2 \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) &= 1 + x e^x \\
\text{ans37} &= y(x) = -4 e^x + 3 x e^x + \frac{1}{6} x^3 e^x - \frac{1}{2} x^2 e^x + x + 4
\end{align*}
\]
```
Part 2: Forced Oscillators:

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> with(plots):
> plot(5*cos(3*t)-5*cos(5*t),t=0..6*Pi,color=black,
> title=`Figure 5.6.2 page 341`);

Figure 5.6.2 page 341

Beating:

If the forcing frequency is not equal to the natural frequency the general solution to will be the superposition of two cos-sin terms, one corresponding to the particular solution with angular frequency \( w \), and the other being the general solution to the homogeneous problem, with angular frequency \( w_0 \).

When \( w \) and \( w_0 \) as well as the corresponding amplitudes are close, the system exhibits **beating**.

Musicians tune their instruments using this phenomenon.

> restart:with(DEtools):with(plots):
> deqtn1a:=diff(x(t),t,t) + w0^2*x(t) = (F0/m)*cos(w*t);
#this is the case w not equal to w0,
#I divided the model equation by m
> dsolve(deqtn1a,x(t));
#general solution, equal to particular
#solution plus general homogeneous eqtn solution.
> sol4:=dsolve({deqtn1a,x(0)=0,D(x)(0)=0},x(t));
#Example 2 page 342
Resonance in undamped, forced harmonic oscillators, when \( w = w_0 \): You can use variation of parameters or undetermined coefficients to solve the forced oscillator, with no damping, in the case that the driving frequency \( w \) exactly equals the natural frequency \( w_0 \). Or, you can use Maple

\[
\text{deqtn1} := \frac{d^2}{dt^2} x(t) + w_0^2 x(t) = \frac{F_0}{m} \cos(w_0 t);
\]

\[
\text{sol1} := x(t) = \sin(w_0 t) C2 + \cos(w_0 t) C1 + \frac{1}{2} \frac{F_0 (\cos(w_0 t) + \sin(w_0 t) w_0 t)}{m w_0^2}
\]

You see there the particular solution which we found, namely \( x_p(t) \):

\[
x_p := t \rightarrow \left( \frac{1}{2} \right) \frac{F_0 t \sin(w_0 t)}{w_0 m}
\]

You can see that \( x_p(t) \) solves the initial value problem \( x_0 = v_0 = 0 \), i.e. the system initially at rest.

For example, here’s how to make the picture on page 343 of the text:

```maple
> F0 := 100; w0 := 50; F0 := 100; m := 1;

> x0 := x(t);

> plot(xp(t), t=0..1.5, color=black, title=`Figure 5.6.4 page 343`);
```