Postpone the last 2 problems in §15.8; #28, 29, until next assignment.

§15.8
Max-min problems
for functions of several variables

Notation

Let $f : D \rightarrow \mathbb{R}$

\[ f(x, y), \ f(x, y, z) \] etc.

$f(p_0)$ is global max. value
if $f(p_0) > f(p) \ \forall p \in D$

global min. value
if $f(p_0) < f(p) \ \forall p \in D$

$f(p_0)$ is a local max or min value
if there is a neighborhood $N$ of $p_0$ so that
$f(p_0)$ is the max or min value of $f$ in $D \cap N$

Theorem: If $D$

is a closed and bounded domain

all points are within some fixed finite distance of origin

And if $f$ is continuous at all points of $D$

Then $f$ attains its global maximum and minimum values. This either happens at boundary points or interior points. If the extreme value is $f$ of an interior point $p_0$ then either
(a) $\nabla f(p_0) = 0$

or
(b) $f$ is not differentiable at $p_0$

Example 1: By inspection above

1. Find all global extrema

2. Find some local max & min values which are not global extrema

3. What's interesting about $(0, 0, 0)$?
Example 2

\[ D = \{ (x,y) \mid 0 \leq x \leq 3, \quad 0 \leq y \leq 3 \} \]

Find the global max and min values of \( f(x,y) = 3x^2 - 6x + 2y^2 - 8y + 13 \)

Example 3

Let \( D = \{ (x,y) \mid x^2 + y^2 \leq 1 \} \)

Find the global max and min values of \( f(x,y) = x^2 + y^2 + 3xy \)
Second derivative test, for \( f(x,y) \):

If \( \nabla f(p) = 0 \)

Compute the Hessian matrix

\[
\begin{bmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{bmatrix}
\]

and let \( D = f_{xx}f_{yy} - f_{xy}^2 \) be its determinant at \( p \)

If

1. \( f_{xx} > 0 \) AND \( D > 0 \) \( f \) is concave up \( \rightarrow f(p) \) local min
2. \( f_{xx} < 0 \) AND \( D > 0 \) \( f \) is CD \( \rightarrow f(p) \) local max
3. \( D < 0 \) \( f(p) \) is a saddle point neither local max or min
4. \( D = 0 \) can't say anything w/o more work

Check Examples 2 & 3 critical pts
with 2nd deriv. test.
Applications (more in HW!)

Example 4

open-topped box made of cardboard
Volume to be 4 m³
What choices of \( x, y, z \) minimize required
surface area of cardboard?

Example 5

Find the nearest point on the paraboloid \( z = x^2 + y^2 \), to \( P = (2,0,0) \).