Math 2210-4
Monday 1/24

- Cross product continued §14.3
- lines and planes §14.4

Recall:
  cross product definition:

  geometric characterization:

Recall:
  We computed
  \[ <2,1,0> \times <1,3,0> = <0,0,5> \]
  using delaminant def.

[1]
  Check (alternate computation)
  \[ (2\hat{i} + \hat{j}) \times (2 + 3\hat{i}) \]
  using algebra properties
  (This was 4 Fri, which we didn’t get to.)

continue with pages 3, 4 Friday,
  including Volumes on page 4.
Example 2

Consider points

\[ P = (1, 0, 3) \]
\[ Q = (0, 1, 1) \]
\[ R = (2, 1, 0) \]

(a) Find the equation of a plane through these 3 points

(b) Find the area of the triangle having \( P, Q, R \) as vertices
Lines in $\mathbb{R}^2$ and in $\mathbb{R}^3$.

In $\mathbb{R}^2$:
- You all know slope-intercept ($y = mx + b$)
- Point-slope ($y - y_0 = m(x - x_0)$)

Ways of writing lines.
(also $ax + by = c$)

There is another way, which works for lines in any dimension, with an easy modification.

Parametric lines
\[
\begin{align*}
x &= x_0 + at \\
y &= y_0 + bt
\end{align*}
\]
(i.e. the parameter $t$ is any real number)

Example 2
Compute and plot the points $(x, y)$, for the indicated $t$-values, for the line
\[
\begin{align*}
x &= 1 + 3t \\
y &= 2 + t
\end{align*}
\]

For any point $P$, the vector $\overrightarrow{OP}$ from the origin to $P$ is called the position vector of $P$.
We often write a parametric line (or curve) by expressing a formula for the position vector $\mathbf{F}(t)$, e.g.
\[
\mathbf{F}(t) = \langle 1, 2 \rangle + t \langle 3, 1 \rangle
\]
for the line in example 2.
We call $\langle 3, 1 \rangle$ the (a) direction vector for this particular line.

Example 3 Add the four position vectors for the points you found in 2.
(Of course, usually when we sketch the line we just sketch the endpoints of the position vectors!!)
Example 4: Find the slope-intercept form of the line in (3) by solving simultaneously for $t$ in each equation.

Lines in $\mathbb{R}^3$

given parametrically by

\[\begin{align*}
  x &= x_0 + at \\
  y &= y_0 + bt \\
  z &= z_0 + ct
\end{align*}\]

or, by expressing the position vector

\[\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle\]

\[\uparrow \quad \text{a point} \quad \uparrow \quad \text{direction vector} \quad \text{on the line}\]

Example 5

Find a parametric expression for the line thru $P = (0,0,1)$ and $Q = (-1,2,2)$

Symmetric form for a line:

Solve for $t$ simultaneously in parametric form:

\[\begin{align*}
  (t =) \quad \frac{x-x_0}{a} &= \frac{y-y_0}{b} = \frac{z-z_0}{c} \\
  \text{equation of a plane} \quad \text{and another plane}
\end{align*}\]

this is expressing the line as an intersection of 2 planes (well, actually of 3 planes)

Example 6: Find symmetric eqts of the line in (5).

Example 7: Find parametric eqts for the line $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z+1}{7}$
Example 3
Find symmetric and parametric equations for the line of intersection between the two planes:

\[ x - y + 2z = 5 \]
\[ 2x + y + z = 4 \]

Lots of ways to do this problem!