Math 2210-1  
Monday 11 Oct

Fri: talked about limits & continuity. 
today: start talking about differentiability

\[ f: \mathbb{R} \to \mathbb{R} \]
\[ f(x,y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h} \quad \text{(hold y fixed, vary in the x-dir.)} \]
\[ f(x,y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{k} \quad \text{(hold x fixed, vary in y-dir.)} \]

"partial derivative in y dir":  
(measures how fast f is changing in y-direction.).

examples: (since only 1 variable is varying, can use 1210 to compute partial derivs!)

\[ f(x,y) = e^{xy} \cos y \]

compute \( \frac{\partial f}{\partial x} \): 
\[ 2f = \frac{\partial f}{\partial x} \]

analogous if more variables:

\[ g(x,y,z) = z^2 \cos(xy^z) \]

define \( \frac{\partial g}{\partial z} \) 

compute \( \frac{\partial g}{\partial z} \)
rates of change interpretation:

Example: The temperature (°C, Centigrade) of a rod of length \( \pi \) (0 ≤ \( x \) ≤ \( \pi \)) is given at time \( t \) by:

\[
T(x, t) = 20(\sin x)e^{-2t}
\]

What is

(a) \( T(\frac{\pi}{2}, 0) \)

(b) how fast is temperature changing at \( x = \frac{\pi}{2} \), when \( t = 0 \)?

(c) how is temperature changing with the \( x \)-variable, at \( x = \frac{\pi}{2} \), \( t = 0 \)?

geometric interpretation:

If \( f : \mathbb{R}^n \to \mathbb{R} \)

we can interpret partial derivatives geometrically in terms of the graph \( z = f(x, y) \):

- \( \frac{\partial f}{\partial y}(x_0, y_0) \) is slope of trace curve \( z = f(x, y) \) in this plane.
- \( \frac{\partial f}{\partial x}(x_0, y_0) \) is slope of trace curve \( z = f(x, y_0) \) in this plane.
Tangent plane to the graph of $f$:

If $f : \mathbb{R} \rightarrow \mathbb{R}^2$ and if the graph of $f$ has a tangent plane at $(x_0, y_0, f(x_0, y_0))$, its equation will be:

$$z = a(x-x_0) + b(y-y_0) + f(x_0, y_0)$$

We want partial derivs to match at $(x_0, y_0)$:

so $a = \frac{\partial f}{\partial x}(x_0, y_0)$

$b = \frac{\partial f}{\partial y}(x_0, y_0)$

Example: for $f(x, y) = x^2 + y^2$

find eqn of tangent plane to graph of $f$, at $(x, y) = (0, 1)$

Sketch.
notation and higher order partial derivs:

\[
\frac{\partial f}{\partial x}, \ f_x \quad \text{same}
\]

\[
\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = (f_x)_y = f_{xy}
\]

\[
\frac{\partial^2 f}{\partial x \partial x} = \frac{\partial^2 f}{\partial x^2} = f_{xx}
\]

etc.

compute all 1st & 2nd order partial derivs for

\[
f(x) = e^x \cos 3y
\]

What do you notice about

\[
f_{xy} \ & f_{yx}?
\]