You can define double integrals for "nice" domains in the plane, and continuous functions defined on these domains.

The "inner partition" for domain D.

In advanced Calculus, one studies the process by which Riemann sums converge to a limit, as the norm of the partitions go to 0.

**How to compute double integrals over non-rectangular domains**

Vertically simple region.

Horizontally simple region.

More complicated region: subdivide.

This makes sense if you think about the volume interpretation of \( \iint f \, dA \). (slicing).
Example

\[ f(x, y) = xy^2 \]

Compute \( \iint_R f(x, y) \, dA \) 2 ways (since region is vertically 2, how simple).

Sketch as you go! (ans = \( \frac{5}{72} \)).
Example

Find the region being integrated over in the following iterated integral. (Also, compute the integral.) Then compute the integral in the other order. (Which can be done!)

\[
\int_{-2}^{1} \left( \int_{2-y}^{2+y} 2x \, dx \right) \, dy.
\]

(ans = \frac{72}{5})
Sometimes you need to reverse the order of integration in order to compute an integral.

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\[ \int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy \]  

(ans: \( \frac{2}{3} (e-1) \))