Problem session room:
   Tuesdays 9:40-10:30
   LCB 121

- vector addition and scalar multiplication (page 2 Wed.)

For \( \vec{u} = \langle u_1, u_2 \rangle \)
\( \vec{v} = \langle v_1, v_2 \rangle \)
\( t \in \mathbb{R} \)

Then
\[
\vec{u} + \vec{v} = \\
\vec{t} \; \vec{u} =
\]

arithmetic properties

\[
\begin{align*}
\vec{u} + \vec{v} &= \vec{v} + \vec{u} & \text{+ is commutative} \\
\vec{u} + (\vec{v} + \vec{w}) &= (\vec{u} + \vec{v}) + \vec{w} & \text{+ is associative} \\
(t \vec{u}) + \vec{v} &= t \vec{u} + \vec{v} & \text{distributive properties} \\
(s + t) \vec{u} &= s \vec{u} + t \vec{u} \\
(st) \vec{u} &= s(t \vec{u}) &= t(s \vec{u}) & \text{- associative}
\end{align*}
\]

Check one of these properties (more on HW):

- page 3 Wed
$\mathbb{R}^3 = \{(x, y, z) \text{ s.t. } x \in \mathbb{R}\}$

$\mathbb{R}^3$ (3-dim Euclidean space)

"is the set of all ordered triples $(x,y,z)$ such that $x$ is an element of the real numbers".

Geometrically, we represent $\mathbb{R}^3$ using right-handed coord systems (actually we used "right-handed" coords for $\mathbb{R}^2$ as well.)

\[\text{your right hand}\]

To see where points are it often helps to plot boxes for which the points are corners.

\textbf{Example}

\textbf{Plot} $(1,2,-3)$

\textbf{Example}

If $P = (x_1, y_1, z_1)$

$Q = (x_2, y_2, z_2)$

then distance from $P$ to $Q$ is

$\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Two applications of Pythagorean Thm?

Details:
Example: (Example 3 p. 782)

What geometric object is described by the set of points satisfying the equation
\( (x, y, z) \)

\[ x^2 + y^2 + z^2 + 4x + 2y - 6z - 2 = 0 ? \]

---

**Vectors in 3-space**

Def. A 3-vector is an ordered triple \( \mathbf{v} = \langle v_1, v_2, v_3 \rangle \) of real numbers.

Def. Length (or magnitude) of \( \mathbf{v} \),

\[ ||\mathbf{v}|| = \sqrt{v_1^2 + v_2^2 + v_3^2} \]

Def. \( \overrightarrow{PQ}, \overrightarrow{OP} \)

Def. If \( \mathbf{u} = \langle u_1, u_2, u_3 \rangle \)

\[ \mathbf{v} = \langle v_1, v_2, v_3 \rangle \]

then \( \mathbf{u} + \mathbf{v} = \mathbf{u} - \mathbf{v} = \mathbf{v} \in \mathbb{R} \)

same properties as geometric interpretation as page 1!

\[ \mathbf{i} = \langle 1, 0, 0 \rangle \]
\[ \mathbf{j} = \langle 0, 1, 0 \rangle \]
\[ \mathbf{k} = \langle 0, 0, 1 \rangle \]

Example: \( \mathbf{a} = \langle 3, 4, 12 \rangle \)
\[ \mathbf{b} = \langle -4, 3, 0 \rangle \]

\( \mathbf{a} + \mathbf{b} = \)

\[ 1\mathbf{a} = \]

\[ 3\mathbf{a} = \]

\[ 3\mathbf{a} - 2\mathbf{b} = \]

Draw a directed line segment representing \( \mathbf{a} \), and illustrate

\[ \mathbf{d} = 3\mathbf{a} + 4\mathbf{j} + 12\mathbf{k} . \]
On Monday we will discuss the dot product of vectors

\[ \langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1v_1 + u_2v_2 + u_3v_3 \quad \text{(a scalar!)}. \]

**Algebraic Properties:**

\[ \bar{u} \cdot u = |\bar{u}|^2 \]
\[ u \cdot \bar{v} = \bar{v} \cdot u \]
\[ u \cdot (\bar{v} + \bar{w}) = u \cdot \bar{v} + u \cdot \bar{w} \]

\[ (\bar{u} + \bar{v}) \cdot \bar{w} = \bar{u} \cdot \bar{w} + \bar{v} \cdot \bar{w} \]

\[ (atu) \cdot \bar{v} = t (u \cdot \bar{v}) = u \cdot (t\bar{v}) \]

**Geometric Property:**

\[ u \cdot \bar{v} = |u||v| \cos \theta \]

---

\[ \text{Let's you find } \cos \theta \text{ algebraically.} \]

**Extremely useful**
Math 2210-1

Homework due 9/1: Hand in circled problems

11.1 1 2 5 6 14 19 24 29 45 51 53
11.2 5 11 17 25 45 50 53 56 57 67 70

And

I. The Pythagorean Theorem says that for a right \( \triangle \), with legs \( a, b \) and hypotenuse \( c \),
\[ c^2 = a^2 + b^2. \]

Use the following diagram (by computing the area of the \((ab) \times (ab)\) square two ways) to prove Pythagoras’ Theorem. Hint: first show the inside \( \square c \) is a square.

![Diagram of Pythagorean Theorem proof]

II. Carefully check one of the 5 arithmetic properties on page 1 of notes, using 3-vectors. Don’t check the one we did in class.

III. Prove the law of cosines for an acute angle \( \theta \) in a triangle, using 2 Pythagorean Thm applications, as indicated by diagram:

\[ c^2 = a^2 + b^2 - 2ab \cos \theta \]

(we use this fact to understand geometric meaning of dot product).

IV. For \( \vec{a} = \langle a_1, a_2, a_3 \rangle \)
\( \vec{b} = \langle b_1, b_2, b_3 \rangle \)
\( \vec{c} = \langle c_1, c_2, c_3 \rangle \)

Show that the dot product distributes over addition:
\[ \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}. \]