The same ground rules will be in effect that we've had for the first two exams. These are typical problems of the sort you may encounter - but make sure to study ALL concepts we've covered.

1) Consider the region bounded between the graph of \( y=x^2 \) and the x-axis, between \( x=0 \) and \( x=2 \).

1a) Use the Fundamental Theorem of Calculus to calculate the exact area of this region. (5 points)

1b) Sketch the region. Include in this sketch the 4 rectangles one obtains by dividing the interval [0,2] into four equal-length subintervals, and using the values of \( f(x)=x^2 \) on the right endpoints for the height of the rectangles. Compute the Riemann sum for this partition, i.e. the sum of these rectangles' areas. (10 points)

1c) Divide the interval [0,2] into \( n \) equal-length subintervals, and for each subinterval choose the right-hand endpoint as your sample point. Write down the resulting Riemann Sum. (5 points)

1d) Using the summation formulas on page 223-224 (these will be provided on the exam), express the Riemann Sum in 1c), in terms of \( n \). (5 points)

1e) Show that the limit as \( n \) approaches infinity of your expression in 1d) equals the exact area you computed in 1a). (5 points)

2) Compute the following: (10 points each, total = 30)

2a) \[
\int_{0}^{2} \frac{t^3}{\sqrt{t^4+9}} \, dt
\]

2b) \[
D_x \int_{0}^{3} \sin(z)^2 \, dz
\]

2c) Solve \[
\frac{dy}{dx} = \frac{6x-x^3}{2y}
\] with \( y=3 \) at \( x=0 \).

3) Consider the region bounded by the two curves \( y=x^2 \) and \( y=-x+2 \).

3a) Sketch the region. Find the coordinates for the intersection points of the two curves. (5 points)

3b) Find the area of the region (5 points)

3c) Consider the solid of revolution obtained by rotating this region about the x-axis. Find its volume. (15 points)

3d) Consider the solid of revolution obtained by rotating this region about the line \( x=2 \). Set up the definite integral which has value equal to its volume. You need not evaluate this integral. (15 points)
1a) \[ \left( \int_0^2 x^2 \, dx = \frac{x^3}{3} \right)_0^2 = \frac{8}{3} \]

1b) \[ R_p = 0.5 \left[ .25 + 1 + 2.25 + 4 \right] = 0.5 \left[ 6.5 \right] = 3.25 \]

1c) \[ \Delta x_i = \frac{2}{n} \]

\[ I_x = \left[ \frac{2}{n}, \frac{2}{n}, \ldots, \frac{2}{n} \right] \quad i = 1, \ldots, n \]

\[ R_p = \sum_{i=1}^{n} f(x_i) \Delta x_i = \sum_{i=1}^{n} \left( \frac{2}{n} \right)^2 \left( \frac{2}{n} \right) \]

1d) \[ \sum_{i=1}^{n} \frac{4x^2}{n} = \frac{8}{n^3} \sum_{i=1}^{n} i^2 = \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} = \frac{4}{3} \frac{n(n+1)(2n+1)}{n^3} \]

1e) \[ \lim_{n \to \infty} \frac{4}{3} \left( \frac{n}{n} \right) \left( \frac{2n+1}{n} \right) = \lim_{n \to \infty} \frac{4}{3} \left( 1 + \frac{1}{n} \right) \left( 2 + \frac{1}{n} \right) = \frac{8}{3} \]

2a) \[ \int_0^2 \frac{t^3}{\sqrt{t^4+9}} \, dt = \int_9^{u^2} \frac{t^3 \, du}{4} = \frac{1}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \bigg|_{9}^{25} = \frac{1}{2} u^{\frac{1}{2}} \bigg|_{9}^{25} = \frac{1}{2} (5-3) = 1 \]
2b) \[ D_x \int h(t) \, dt = h(g(x))g'(x) - h(f(x))f'(x), \text{ so} \]

\[ D_x \int (\sin^2 x)^2 \, dx = (\sin(3x))^2 \cdot 3 \]

2c) \[ 2y \, dy = (6x-x^3) \, dx \]

\[ \int \frac{y^2}{3} = \frac{3x^2}{9} - \frac{x^4}{4} + C \quad \int \frac{y^2}{4} = \frac{3x^2}{8} - \frac{x^4}{4} + 9 \]

\[ y(0) = 3: \quad 9 = 0 \cdot 0 + C \quad C = 9 \]

\[ y = \sqrt{3x^2 - \frac{x^4}{4} + 9} \]

(notice vertical scale is compressed relative to horizontal scale)

3a) \[ y = -x + 2 \]

3b) \[ dA = \left[ (-x+2) - x^2 \right] \, dx \]

\[ A = \int_{-2}^{1} \left( -x+2 - x^2 \right) \, dx = \left[ -\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-2}^{1} = -\frac{1}{2} + 2 - \frac{1}{3} - \left[ -2 - \frac{4}{3} \right] = \frac{8}{3} - \frac{1}{2} - \frac{3}{2} = 4\frac{1}{3} \]

3d) \[ V = \pi \left( (-x+2)^2 - x^2 \right) \, dx = \pi \left( -x^4 + x^2 - 4x + 4 \right) \, dx \]

\[ V = \int_{-2}^{2} \pi \left( x^4 + x^2 - 4x + 4 \right) \, dx = \pi \left[ -\frac{x^5}{5} + \frac{x^3}{3} - 2x^2 + 4x \right]_{-2}^{2} \]

\[ V = \pi \left[ -\frac{1}{5} (1-5) + \frac{1}{3} (1+5) - 2 (1-4) + 4 (1-2) \right] \]

\[ V = \pi \left[ \frac{2}{5} + 3 + 6 + 12 \right] = \pi \left( 27 \cdot \frac{2}{5} \right) \]
Math 1210-3/4
Wed 9 April

Review for 3rd midterm Friday
• 2nd page is practice exam. At least 80% of the exam topics are represented there. Solutions will be posted tomorrow, and the Thursday problem sessions will be exam review sessions.
   Thurs 9:40-10:30 OSH Auditorium
   11:50-12:40

• Exam covers 5.1-6.3
  antidifferentiation
  polynomials
  powers
  trig fns
  substitution
  separable differential eqns
  definite integrals with FTC
  substitution
  FTC 1 & 2
  properties of integration (linearity, interval additivity)
  Riemann sums
  for specific partition
  for equi-partition of n subintervals
  \[ \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \int_a^b f(x) \, dx \text{ for } a \leq x \leq b \]
  actual limit computation for equi-partition
  (summation formulas p223-224 will be provided)

Applications
  area
  volumes by slicing (slabs, disks, washers)
  volumes of revolution by cylindrical shells
  average value

• In addition to practice exam consider old HW's & chapter reviews p266-267
  p312-313