1) Here is the graph of a mystery function $f(x)$:

Consider the four related functions below, and plot their graphs onto the picture above. Indicate clearly which graph corresponds to which function, and explain how you obtained it from the graph of $f$. (5 points each, for 20 total points)

1a) $g(x) = f(x) + 1$
   
   **shift up 1 unit**

1b) $h(x) = f(x + 1)$
   
   **move left 1 unit**

1c) $k(x) = \frac{1}{2} f(x)$
   
   **scale vertically by factor of $\frac{1}{2}$**

1d) $m(x) = f(2x)$
   
   **compress horizontally by a factor of 2**
2) Here is a plot of the population of the United States from 1800 to 1900. The vertical axis is marked off in millions of people.

2a) Use the graph to estimate the average population growth rate between 1820 and 1880. Include correct units!

\[
\frac{\Delta P}{\Delta t} = \frac{P(1880) - P(1820)}{1880 - 1820} \approx \frac{70 - 10}{60} \times 10^6 \text{ people years} \\
= \frac{3}{3} \times 10^6 \text{ people/year} \\
\approx 670,000 \text{ people/year} \tag{10 points}
\]

2b) Use the graph to estimate the (instantaneous) population growth rate in 1880.

\[
\approx (70 - 0) \times 10^6 \text{ people} \\
1880 - 1880 \text{ years} \\
= 1.2 \times 10^6 \text{ people/year} \\
= 1,200,000 \text{ people/year} \tag{10 points}
\]

3) Compute the following derivatives (25 points total)

3a) \[D_t (t^2 + 8t^2 - 7) = 3t^2 + 16t \tag{8 points}\]

3b) \[D_x y = \frac{d}{dx} [\sin(3x)](x^2 + 1) \]
\[= (\cos(3x))3(x^2 + 1) + (\sin(3x))3x^2 \tag{8 points}\]

3c) \[D_x y = \frac{(2x+1)^2}{4x^2 - 7x} \]
\[= \frac{5'g - 5g'}{g^2} \tag{9 points}\]
\[= \frac{2(2x+1)2(4x^2 - 7x) - (2x+1)^2(12x^2 - 7)}{(4x^2 - 7x)^2} \]
4a) Use the limit definition of derivative, 
\[ \frac{f(x)}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
to compute the derivative of \( f(x) = \frac{1}{2x + 3} \).

\[ f'(x) = \lim_{h \to 0} \frac{1}{h} \left[ \frac{(2x+3) - (2x+3)}{2(x+1)^3} \right] 
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{2x+3}{2(x+1)^3} \right] = \frac{1}{(2x+3)^2} \]

(15 points)

5) True/False. No justification needed.

(a) The derivative of a polynomial is always a polynomial, \( \frac{f(x)}{h} = \frac{1}{h} \).

(b) The addition angle formula for cosine is: \( \cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \)

(c) For the number \( \pi \), which is the area of the unit disk, the derivative of the function \( f(x) = \pi^2 \)
is \( D_x f(x) = 4\pi^2 \), \( f(x) = \pi \) is a constant so \( D_x f(x) = 0 \)

(d) If a function is continuous at a point, then it must also be differentiable there.

(e) If \( f(x) \) has derivative \( f'(x) = 3 \), and value \( f(2) = 5 \), then the derivative of the function \( f(x)^4 \) at \( x = 2 \) is 1600.

\[ D_x f(x)^4 = 4 f(x)^3 f'(x) \]

\[ x = 2, \quad 4 (5^3)(3) = 1500 \]

4b) Check your answer in part (4a) by using your favorite differentiation rules to compute the derivative of \( f(x) = \frac{1}{2x + 3} \).

\[ f'(x) = (2x + 3)^{-1} \]

(5 points)

4c) Find the equation of the tangent line to the graph of \( y = \frac{1}{2x + 3} \) passing through the point with \( x \)-coordinate equal to 1.

\[ \tan \theta = f'(1) = \frac{1}{2} \]

(5 points)