Practice Problems for Sections 4.2 and 4.3 – MATH 1170, Fall 2007

1. (a) Solve the differential equation \( \frac{dp}{dt} = 4t + \frac{3}{t^2} \), \( p(1) = -1 \).
(b) What happens to the solution around \( t = 0 \)?
(c) Where does \( p(t) = 17 \)?
(d) Sketch the solution \( p(t) \) for the domain \( 0 \leq t \leq 3 \).

2. Use substitution to find the integrals of the following functions.
   (a) \( \frac{1}{(1+5t)^2} \)
   (b) \( \frac{2}{x}(\ln(x))^2 \)
   (c) \( 6x(1+x^2)^2 \)
   (c) \( x^2e^{x^3} \)

3. Use integration by parts to evaluate the following functions.
   (a) \( \int \ln(x)dx \)
   (b) \( \int x^3e^{x^2}dx \) (Hint: Also use substitution here \( y = x^2 \))

4. (a) Solve the differential equation \( \frac{dM}{dt} = te^{-t} \), \( M(0) = 1 \). It represents the rate of change of a fungal population in a Petri dish, used for medical purposes.
   (b) As \( t \) gets large, what does \( M(t) \) tend to?
   (c) Why would \( M(t) \) behave like this as \( t \) gets large?