Practice Exam

1) Consider the differential equation
\[ \frac{dP}{dt} = -P^2 + 4P - 3 \]
which models a certain population problem.

a) Find the equilibrium solutions.

b) Sketch the slope field for this differential equation. Onto the slope field sketch graphs of the solutions to the three initial value problems with \( P(0) = 0 \), \( P(0) = 2 \), \( P(0) = 4 \). (You don't need formulas for the solutions to make the sketches!)

c) Which of the equilibrium solutions are stable? Which are unstable?

d) Give a population model which leads to differential equations of this type. Be as precise as you can.

e) Find a explicit solution to the initial value problem for this differential equation, with \( P(0) = 2 \). Verify that your limiting population agrees with what your sketch predicted in part 1b).

2) Consider a brine tank which holds 15,000 gallons of continuously-mixed liquid. Let \( x(t) \) be the amount of salt (in pounds) in the tank at time \( t \). The in-flow and out-flow rates are both 150 gallons/hour, and if the concentration of salt flowing in is 1 pound per 10 gallons of water.

2a) Explain how the information above leads to the differential equation
\[ \frac{dx}{dt} + 0.01x = 15 \]

b) Solve the initial value problem for this differential equation, assuming that at time \( t=0 \) there is no salt in the water. You may use either chapter 1 or chapter 3 techniques.

c) What is the limiting amount of salt as \( t \) approaches infinity?