12. 
   a. \( P(A \cup B) = .50 + .40 - .25 = .65 \)
   b. \( P(A \cup B)' = 1 - .65 = .35 \)
   c. \( A \cap B' \cap (A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25 \)

13. 
   a. awarded either #1 or #2 (or both):
      \( P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36 \)
   b. awarded neither #1 or #2:
      \( P(A_1' \cap A_2') = P[(A_1 \cup A_2)'] = 1 - P(A_1 \cup A_2) = 1 - .36 = .64 \)
   c. awarded at least one of #1, #2, #3:
      \( P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = .22 + .25 + .28 - .11 - .05 - .07 + .01 = .53 \)
   d. awarded none of the three projects:
      \( P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47 \)
   e. awarded #3 but neither #1 nor #2:
      \( P(A_3' \cap A_1' \cap A_2') = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = .28 - .05 - .07 + .01 = .17 \)

19. Let event \( A \) be that the selected joint was found defective by inspector A. \( P(A) = \frac{744}{10000} \). Let event \( B \) be analogous for inspector B. \( P(B) = \frac{551}{10000} \). Compound event \( A \cup B \) is the event that the selected joint was found defective by at least one of the two inspectors. \( P(A \cup B) = \frac{1159}{10000} \).
   
   a. The desired event is \( (A \cup B)' \), so we use the complement rule:
      \( P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{1159}{10000} = \frac{3441}{10000} = .3441 \)
   
   b. The desired event is \( B \cap A' \). \( P(B \cap A') = P(B) - P(A \cap B) \).
      \( P(A \cap B) = P(A) + P(B) - P(A \cup B) = .0724 + .0751 - .1159 = .0316 \)
      So \( P(B \cap A') = P(B) - P(A \cap B) = .0751 - .0316 = .0435 \)
21.  

a.  \( P(\{M,H\}) = .10 \)

b.  \( P(\text{low auto}) = P(\{L,N, (L,L), (L,M), (L,H)\}) = .04 + .06 + .05 + .03 - .18 \) Following a similar pattern, \( P(\text{low homeowner's}) = .06 + .10 + .03 = .19 \)

c.  \( P(\text{same deductible for both}) = P(\{LL, MM, HH\}) = .06 + .20 + .15 = .41 \)

d.  \( P(\text{deductibles are different}) = 1 - P(\text{same deductibles}) = 1 - .41 = .59 \)

e.  \( P(\text{at least one low deductible}) = P(\{LN, LL, LM, LH, ML, HL\}) = .04 + .06 + .05 + .03 + .10 + .03 = .31 \)

f.  \( P(\text{neither low}) = 1 - P(\text{at least one low}) = 1 - .31 = .69 \)

23.  

Assume that the computers are numbered 1 – 6 as described. Also assume that computers 1 and 2 are the laptops. Possible outcomes are (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

a.  \( P(\text{both are laptops}) = P(\{1,2\}) = \frac{1}{15} = .067 \)

b.  \( P(\text{both are desktops}) = P(\{3,4, 3,5, 3,6, 4,5, 4,6, 5,6\}) = \frac{6}{15} = .40 \)

c.  \( P(\text{at least one desktop}) = 1 - P(\text{no desktops}) = 1 - P(\text{both are laptops}) = 1 - .067 = .933 \)

d.  \( P(\text{at least one of each type}) = 1 - P(\text{both are the same}) = 1 - P(\text{both laptops}) - P(\text{both desktops}) = 1 - .067 - .40 = .533 \)
52.

a. \[ 5 \times 4 \times 3 \times 4 = 240 \]

b. \[ 1 \times 1 \times 3 \times 4 = 12 \]

c. \[ 4 \times 3 \times 3 \times 3 = 108 \]

d. \# with at least one Sony = total \# - \# with no Sony = 240 - 108 = 132

e. \[ P(\text{at least one Sony}) = \frac{132}{240} = .55 \]

\[ P(\text{exactly one Sony}) = P(\text{only Sony is receiver}) + P(\text{only Sony is CD player}) + P(\text{only Sony is deck}) \]

\[ = \frac{1 \times 3 \times 3 \times 3}{240} + \frac{4 \times 1 \times 3 \times 3}{240} + \frac{4 \times 3 \times 3 \times 1}{240} = \frac{27 + 36 + 36}{240} \]

\[ = \frac{99}{240} = .413 \]

b. \[ P(\text{none selected}) = 1 - P(A \cup B \cup C) = 1 - .98 = .02 \]

c. \[ P(\text{only automatic transmission selected}) = .03 \text{ from the Venn Diagram} \]

d. \[ P(\text{exactly one of the three}) = .03 + .08 + .13 = .24 \]

33.

a. \[ \binom{25}{5} = \frac{25!}{5!20!} = 53,130 \]

b. \[ \binom{8}{4} \cdot \binom{17}{1} = 1190 \]

c. \[ P(\text{exactly 4 have cracks}) = \frac{\binom{8}{4} \binom{17}{1}}{\binom{25}{5}} = \frac{1190}{53,130} = .022 \]

d. \[ P(\text{at least 4}) = P(\text{exactly 4}) + P(\text{exactly 5}) \]

\[ = \frac{\binom{8}{4} \binom{17}{1}}{\binom{25}{5}} + \frac{\binom{8}{5} \binom{17}{0}}{\binom{25}{5}} = .022 + .001 = .023 \]
35. There are 10 possible outcomes – \( \binom{5}{2} \) ways to select the positions for B's votes: BAAAA, BABAA, BAAAB, ABAAA, ABABA, AABAB, AABAB, AABAB, and AAABB. Only the last two have A ahead of B throughout the vote count. Since the outcomes are equally likely, the desired probability is \( \frac{2}{10} = .20 \).

39.

a. We want to choose all of the 5 cordless, and 5 of the 10 others, to be among the first 10 serviced, so the desired probability is

\[
\binom{5}{5} \binom{10}{5} \binom{15}{10} = \frac{252}{3003} = .0839
\]

b. Isolating one group, say the cordless phones, we want the other two groups represented in the last 5 serviced. So we choose 5 of the 10 others, except that we don't want to include the outcomes where the last five are all the same.

So we have \( \binom{10}{5} - 2 \). But we have three groups of phones, so the desired probability is

\[
3 \cdot \frac{\binom{10}{5} - 2}{\binom{15}{5}} = \frac{3(250)}{3003} = .2498.
\]

c. We want to choose 2 of the 5 cordless, 2 of the 5 cellular, and 2 of the corded phones:

\[
\binom{5}{2} \binom{5}{2} \binom{5}{2} = \frac{1000}{5005} = .1998
\]
40.

a. If the A's are distinguishable from one another, and similarly for the B's, C's and D's, then there are 12! Possible chain molecules. Six of these are:
   \[ A_1A_2A_3B_1B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]
   \[ A_1A_2B_1A_3B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]
   \[ A_1A_2B_1A_3B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]
   \[ A_1A_2B_1A_3B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]
   \[ A_1A_2B_1A_3B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]
   \[ A_2A_3A_1B_1B_2C_1C_2C_3C_4D_1D_2D_3D_4B_1B_2B_3 \]

   These 6 (-3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable). At this point there are \( \frac{12!}{3!} \) molecules. Now suppressing subscripts on the B's, C's and D's in turn gives ultimately \( \frac{12!}{3!} = 369,600 \) chain molecules.

b. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! Ways to order these entities, and thus 4! Molecules in which the A's are contiguous, the B's, C's, and D's are also. Thus, \( P(\text{all together}) = \) \( \frac{4!}{3!} = .00006494 \).

41.

a. \( P(\text{at least one F among 1st 3}) = 1 - P(\text{no F's among 1st 3}) \)
\[ = 1 - \frac{4 \times 3 \times 2}{8 \times 7 \times 6} = 1 - \frac{24}{336} = 1 - .0714 = .9286 \]

An alternative method to calculate \( P(\text{no F's among 1st 3}) \) would be to choose none of the females and 3 of the 4 males, as follows:
\[ \binom{4}{0} \binom{4}{3} = \frac{4}{56} = .0714 \], obviously producing the same result.

b. \( P(\text{all F's among 1st 5}) = \binom{4}{4} \binom{4}{1} = \frac{4}{56} = .0714 \)

c. \( P(\text{orderings are different}) = 1 - P(\text{orderings are the same for both semesters}) \)
\[ = 1 - \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) \times (8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)} = .99997520 \]
44. \[ \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!k!} = \binom{n}{n-k} \]

The number of subsets of size k - the number of subsets of size n-k, because each subset of size k corresponds exactly one subset of size n-k (the n-k objects not in the subset of size k).

47.

a. \( P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{.25}{.50} = .50 \)

b. \( P(B' \mid A) = \frac{P(A \cap B')}{P(A)} = \frac{.25}{.50} = .50 \)

c. \( P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.40} = .6125 \)

d. \( P(A' \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.15}{.40} = .3875 \)

e. \( P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{.50}{.65} = .7692 \)
49. The first desired probability is \( P(\text{both bulbs are 75 watt} \mid \text{at least one is 75 watt}) \).
\[ P(\text{at least one is 75 watt}) - 1 - P(\text{none are 75 watt}) \]
\[ -1 \cdot \binom{9}{2} = 1 - \frac{36}{105} = \frac{69}{105}. \]

Notice that \( P(\text{(both are 75 watt)}) \cdot P(\text{(at least one is 75 watt)}) \)
\[ - P(\text{both are 75 watt}) = \binom{6}{2} = \frac{15}{105}. \]

So \( P(\text{both bulbs are 75 watt} \mid \text{at least one is 75 watt}) = \frac{15}{105} = \frac{15}{69} = .2174 \).

Second, we want \( P(\text{same rating} \mid \text{at least one NOT 75 watt}) \).
\[ P(\text{at least one NOT 75 watt}) - 1 - P(\text{both are 75 watt}) \]
\[ -1 \cdot \frac{15}{105} = \frac{90}{105}. \]

Now, \( P(\text{same rating} \mid \text{at least one not 75 watt}) = P(\text{both 40 watt or both 60 watt}). \)
\[ P(\text{both 40 watt or both 60 watt}) = \binom{4}{2} + \binom{5}{2} = \frac{16}{105} \]

Now, the desired conditional probability is \( \frac{16}{90} = \frac{16}{90} = .1778 \).

51.

a. \( P(R \text{ from 1st } \cap R \text{ from 2nd}) = P(R \text{ from 2nd} \mid R \text{ from 1st}) \cdot P(R \text{ from 1st}) \)
\[ = \frac{8}{11} \cdot \frac{6}{10} = .436 \]

b. \( P(\text{same numbers}) = P(\text{both selected balls are the same color}) \)
\[ = P(\text{both red}) + P(\text{both green}) = \frac{4}{11} \cdot \frac{4}{10} = .581 \]
55.

a. \[
P(\text{A and B}) - P(B|A) \cdot P(A) = \frac{2 \times 1}{4 \times 3} \times \frac{2 \times 1}{6 \times 5} = .0111
\]

b. \[
P(\text{two other H’s next to their wives | J and M together in the middle})
\]
\[
\frac{P[(H - W, \text{ or } W - H) \text{ and } (J - M, \text{ or } M - J) \text{ and } (H - W, \text{ or } W - H)]}{P(J - M, \text{ or } M - J \text{ in the middle})}
\]
\[
\text{numerator } = \frac{4 \times 1 \times 2 \times 1 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{16}{6!}
\]
\[
\text{denominator } = \frac{4 \times 3 \times 2 \times 1 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{48}{6!}
\]
so the desired probability = \[\frac{16}{48} = \frac{1}{3}\].

59.

\[
.A \times .3 = .12 = P(A_1 \cap B) = P(A_1) \cdot P(B | A)
\]
\[
.35 \times .6 = .21 = P(A_2 \cap B)
\]
\[
.25 \times .5 = .125 = P(A_3 \cap B)
\]

a. \[
P(A_2 \cap B) = .21
\]

b. \[
P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455
\]

c. \[
P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.12}{.455} = .264
\]
\[
P(A_2|B) = \frac{.21}{.455} = .462, P(A_3|B) = 1 - .264 - .462 = .274
\]
a. Since the events are independent, then \( A' \) and \( B' \) are independent, too. (see paragraph below equation 2.7. \( P(B'|A') = \cdot \cdot .7 - .3 \)

b. \( P(A \cup B) = P(A) + P(B) - P(A)P(B) = .4 + .7 + (.4)(.7) = .82 \)

c. \( P(AB'|A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB')}{P(A \cup B)} = \frac{.12}{.82} = .146 \)

\[ \begin{align*}
\text{reg} & \quad .35 \\
\text{extra} & \quad .25 \\
\text{prem} & \quad .10
\end{align*} \]

\[ \begin{align*}
\text{fill} & \quad .5 \\
\text{no fill} & \quad .5
\end{align*} \]

\[ \begin{align*}
\text{credit} & \quad .0700 \\
\text{credit} & \quad .0625 \\
\text{credit} & \quad .0500
\end{align*} \]

a. \( P(U \cap F \cap C) = .1260 \)

b. \( P(Pr \cap NF \cap C) = .05 \)

c. \( P(Pr \cap C) = .0625 + .05 = .1125 \)

d. \( P(F \cap C) = .0840 + .1260 + .0625 = .2725 \)

e. \( P(C) = .5325 \)

f. \( P(PR|Cr) = \frac{P(Pr \cap C)}{P(C)} = \frac{.1125}{.5325} = .2113 \)
P(satis) = .51

P(mean | satis) = \frac{.2}{.51} = .3922

P(median | satis) = .2941

P(mode | satis) = .3137

So Mean (and not Model) is the most likely author, while Median is least.
a. \( P(U \cap F \cap Cr) = .1260 \)

b. \( P(Pr \cap NF \cap Cr) = .05 \)

c. \( P(Pr \cap Cr) = .0625 + .05 = .1125 \)

d. \( P(F \cap Cr) = .0840 + .1260 + .0625 = .2725 \)

e. \( P(Cr) = .5325 \)

f. \( P(Pr \mid Cr) = \frac{P(Pr \cap Cr)}{P(Cr)} = \frac{.1125}{.5325} = .2113 \)
71. \[ P(A' \cap B) = P(B) - P(A \cap B) = P(B) - P(A) \cdot P(B) - [1 - P(A)] \cdot P(B) = P(A') \cdot P(B) \]

Alternatively, \[ P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} \]

\[ = \frac{P(B) - P(A) \cdot P(B)}{P(B)} = 1 - P(A) = P(A') \]

15. Let \( q \) denote the probability that a rivet is defective.

\( a. \quad P(\text{seam need rework}) = .20 \cdot (1 - P(\text{seam doesn’t need rework})) \]
\[ = 1 - P(\text{no rivets are defective}) \]
\[ = 1 - P(1^{\text{st}} \text{ isn’t def} \cap \ldots \cap 25^{\text{th}} \text{ isn’t def}) \]
\[ = 1 - (1 - q)^{25}, \text{ so } .80 = (1 - q)^{25}, 1 - q = (.80)^{1/25}, \text{ and thus } q = 1 - .99111 = .00889. \]

\( b. \quad \text{The desired condition is } .10 = 1 - (1 - q)^{25}, \text{ i.e. } (1 - q)^{25} = .90, \text{ from which } q = 1 - .99579 \approx .00421. \]

78. \[ P(\text{system works}) = P(1 - 2 \text{ works } \cup 3 - 4 \text{ works}) \]
\[ = P(1 - 2 \text{ works}) + P(3 - 4 \text{ works}) - P(1 - 2 \text{ works } \cap 3 - 4 \text{ works}) \]
\[ = P(1 \text{ works } \cup 2 \text{ works}) + P(3 \text{ works } \cap 4 \text{ works}) - P(1 - 2 \text{ works}) \cdot P(3 - 4) \]
\[ = (.9 + .9 - .81) + (.9)(.9) - (.9 + .9 - .81)(.9)(.9) \]
\[ = .99 + .81 - .8019 = .9981 \]

87.

\[
\begin{align*}
\pi^2 & \quad c_2
\
\pi (1 - \pi) & \quad c_1
\
1 - \pi & \quad \pi
\end{align*}
\]

\[ P(\text{at most 1 is lost}) = 1 - P(\text{both lost}) \]
\[ = 1 - \pi^2 \]

\[ P(\text{exactly 1 lost}) = 2\pi(1 - \pi) \]

\[ P(\text{exactly 1 at most 1}) = \frac{P(\text{exactly 1})}{P(\text{at most 1})} = \frac{2\pi(1 - \pi)}{1 - \pi^2} \]